A ONE-DIMENSIONAL SUBGRID NEAR-WALL TREATMENT FOR REYNOLDS AVERAGED COMPUTATIONAL FLUID DYNAMICS SIMULATIONS

By

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A ONE-DIMENSIONAL SUBGRID NEAR-WALL TREATMENT FOR 
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Prediction of the near wall region is crucial to the accuracy of turbulent flow computational fluid dynamics (CFD) simulation. However, sufficient near-wall resolution is often prohibitive for high Reynolds number flows with complex geometries, due to high memory and processing requirements. A common approach in these cases is to use wall functions to bridge the region from the first grid node to the wall. This thesis presents an alternative method that relaxes the near wall resolution requirement by solving one dimensional transport equations for velocity and turbulence across a locally defined subgrid contained within wall adjacent grid cells. The addition of the subgrid allows for wall adjacent primary grid sizes to vary arbitrarily from low-Re model sizing ($y^+\approx 1$) to wall function sizing without significant loss of accuracy or increase in computational cost.
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NOMENCLATURE

$C_f$ Skin friction coefficient

$E$ Wall function coefficient

$K$ Primary grid turbulent kinetic energy

$k$ Subgrid turbulent kinetic energy

$k^+$ Dimensionless turbulent kinetic energy

$P$ Pressure

$Re$ Downstream Reynolds number measured from flat plate leading edge

$Re_y$ Wall Reynolds number

$S$ Strain rate magnitude

$S_{ij}$ Strain rate tensor

$S_v$ Vorticity magnitude

$\tilde{S}_v$ Modified vorticity magnitude

$U, V$ Primary grid tangential and wall normal velocity

$U_j$ Primary grid velocity vector

$\overline{U_i U_j}$ Primary grid kinematic Reynolds stress tensor
\( u, v \) Subgrid tangential and wall-normal velocity

\( \overline{u'v'} \) Subgrid kinematic Reynolds shear stress

\( u' \) Dimensionless tangential velocity

\( u_f \) Wall friction velocity

\( v' \) Dimensionless normal velocity

\( x,y \) Tangential and wall normal coordinates

\( Y \) Wall distance of first primary grid node

\( y^+ \) Dimensionless wall distance

\( \varepsilon \) Turbulence dissipation rate

\( \gamma \) Near-wall mesh stretching ratio

\( \kappa \) Von Karman constant

\( \mu \) Molecular dynamic viscosity

\( \mu_t \) Turbulent (eddy) viscosity

\( \mu_e \) Effective viscosity

\( \Omega_{ij} \) Rate of rotation tensor

\( \rho \) Density

\( \tau_w \) Wall shear stress

\( \nu^* \) Turbulent kinematic viscosity
\( \chi \)        Viscosity ratio 

Quantities for the Wolfshtein Turbulence Model:

\( l_D \)        turbulence dissipation length scale 

\( l_\mu \)        turbulent viscosity length scale 

\( P_k \)        turbulent kinetic energy production 

\( \sigma \)        turbulent diffusivity coefficient 

Quantities for the Spalart-Allmaras Turbulence Model:

\( f_w \)        wall damping function 

\( f_{vl} \)        viscous damping function 

\( f_{v2} \)        viscous damping function 

\( G_v \)        production of turbulent viscosity 

\( \sigma_v \)        turbulent diffusivity coefficient 

Subscripts:

\( i \)        Subgrid index, when used on subgrid variable 

\( n \)        Subgrid index ahead of current index \((i + 1)\) 

\( s \)        Subgrid index behind current index \((i - 1)\)
CHAPTER I

INTRODUCTION

Many practical problems in engineering fluid mechanics involve turbulent flow near a solid wall. Of particular importance in these problems is the solid-fluid interface at which free stream turbulent fluctuations vanish due to the no-slip condition. Immediately adjacent to the wall, momentum transfer is dominated by viscosity and the resulting near-wall flow is distinct from the free stream flow in which inertial effects prevail. Several methods have been developed for computational fluid dynamics (CFD) simulation that account for this shift between viscous and inertial dominance in the boundary layer with varying degrees of success. This paper deals with a new method to treat near-wall behavior in CFD.

The objective of the present study is to further increase the flexibility of subgrid-based near-wall treatments by reducing the near-wall computations to a locally one-dimensional representation. The source terms for the primary grid solution are obtained in the near-wall cells by numerically solving the 1-D ordinary differential equations for tangential momentum and turbulence model quantities. The resulting near wall treatment allows arbitrary mesh sizing of the first layer cell, with accuracy levels and computational cost comparable to either a low-Re or wall function approach. The new method
incorporates less empiricism than hybrid wall functions and does not require databases or lookup tables.
CHAPTER II
NEAR WALL TREATMENT

2.1 Low-Reynolds-Number Models

Low-Reynolds-number models have been used successfully to bridge the gap between free stream and boundary layer flow while honoring the physics of the problem. Standard low-Reynolds-number models range from the simple mixing length model to two and three equation non-linear models. Regardless of the type, these models supply equations that are applicable from the free stream down to the wall. Consequently, a fine near-wall grid relative to the free stream is required to capture the large wall normal gradients in the boundary layer.

Low-Reynolds-number models can produce accurate solutions with proper near-wall gridding which places a large percentage of cells in the boundary layer. However, the obvious trade off is a significant increase in computational time due to these near-wall requirements. Despite the relatively small size of the boundary layer compared to the overall flowfield, high cell stretching and aspect ratios, as well as steep gradients of flow variables, cause the solution in this region to progress slower than that of the free stream where cells are typically more uniform. The low-Reynolds-number models are therefore
not amenable to complex geometries when computational resources are limited. Under these circumstances, another method is often desired which does not impose as high a computational cost.

2.2 The Wall Function Method

A popular alternative to the low-Reynolds-number approach is to use wall functions, which replace the near-wall difference equations with algebraic equations that can be solved on a larger near-wall mesh and still capture the effects of the boundary layer \[1\][2]. Consequently, wall functions yield substantially reduced computational times over the low-Reynolds-number approach. However, wall functions typically assume a semi-logarithmic boundary layer velocity profile and rely on experimental or other numerical results to determine an algebraic correlation. This method works well for flows that do not separate or suffer large pressure gradients but often becomes inaccurate when conditions deviate significantly from the correlated conditions. A number of wall functions have been proposed that utilize more complex analytical prescriptions of the near-wall variable profiles \[3\][4][5][6], which are intended to improve their accuracy under complex flow conditions, yet all are based on an assumed semi-logarithmic profile and are inherently empirical.

In an attempt to increase the flexibility and decrease the mesh sensitivity of wall functions, hybrid approaches have been proposed that use piecewise or blended profiles for different regions of the turbulent boundary layer \[7\][8]. These approaches, while still based on algebraic prescription of variable profiles, allow reasonable results to be obtained for first node locations in the viscous sublayer, buffer region, or inertial
sublayer, therefore relaxing the restriction on near-wall refinement inherent with “pure” wall functions.

An alternative to algebraic profiles is the use of a database of velocity and turbulence data obtained from a fine mesh simulation using the low-Re formulation of a given turbulence model [9]. The simulation uses the database as a “lookup table” in order to prescribe variables in the first cell. This approach yields an accurate prescription of the near-wall velocity field that would be obtained with the low-Re model, but the database is obtained under a limited set of assumed flow conditions (e.g. zero-pressure gradient boundary layer). Similar to hybrid wall functions, however, this method does allow more flexibility in meshing of the near-wall region.

2.3 The Immerged Subgrid Method

Craft et al. [10] recently proposed a technique based on separate resolution of the near-wall region with an imbedded subgrid. In their approach, a 2-D subgrid is defined in the region of the flow occupied by the first layer of wall adjacent primary grid cells. The 2-D boundary layer equations are solved on the subgrid for tangential momentum and turbulence quantities. The streamwise pressure gradient is assumed uniform within each near-wall primary grid cell, thus decoupling the velocity and pressure field solution and eliminating the need to solve the pressure correction equation within the subgrid. The 2-D continuity equation is used to specify the wall-normal velocity component at each subgrid cell. The solution obtained on the subgrid is used to calculate source terms for the primary grid solution, including wall shear stress and turbulence production and dissipation rate in the first layer cells.
The 2-D subgrid approach was shown to yield results comparable to the low-Re model with a well-resolved near-wall mesh [10]. The subgrid approach was also shown to provide a reduction in computing time versus the low-Re approach – up to an order of magnitude – primarily due to a substantial reduction in the number of iterations required for convergence of comparable simulations. Comparison with a standard wall function approach on an identical primary grid indicated that the subgrid approach only required a net increase in CPU time of between 60-120% for the cases tested. However, the 2-D methodology still requires computation of the convective terms in the subgrid solution. This constraint significantly limits the flexibility of the approach, for example near corner cells in complex geometries, since wall fluxes and turbulence source terms cannot be computed independently in each near-wall primary grid cell.
CHAPTER III
THE ONE-DIMENSIONAL SUBGRID METHOD

3.1 Formulation

In the one-dimensional subgrid (1DS) method developed herein, the primary mesh is augmented by a 1-D subgrid applied within each of the near-wall cells to facilitate resolution of this region. The subgrid extends from the wall to the first primary grid node (cell centroid) as illustrated in Figure 3.1. The subgrid utilizes a simplified set of differential equations to calculate wall shear stress and turbulence production, which are used as source terms in the primary grid solution. The details of the formulation are presented below for the case of incompressible, isothermal flow.

3.1.1 Momentum Equation

The velocity in the near-wall region is decomposed into a wall-parallel (tangential) component and a wall-normal component. The tangential momentum is assumed to be governed by the 2-D boundary layer equation:

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \frac{\partial (\rho u' v')}{{\partial y}} ,
\]  

(3.1)
Figure 3.1: Illustration of near-wall subgrid extending from the wall to the first primary grid node.
where $\rho u'v'$ is the turbulent shear stress obtained from the subgrid solution of the turbulence model equations. It is further assumed that the tangential pressure gradient is uniform in the wall-normal direction and equal to the pressure gradient obtained from the primary grid solution in the first near-wall cell:

$$\frac{\partial p}{\partial x} = \frac{\partial P}{\partial x}.$$  

Note that subgrid and primary grid variables are denoted by lower case and upper case symbols, respectively.

The 1-D solution of Eq. (3.1) requires that both the streamwise velocity gradient, $\frac{\partial u}{\partial x}$, and the wall-normal velocity, $v$, be prescribed as a function of the wall distance. The current approach makes use of the 2-D continuity equation:

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y},$$

so that only the prescription of $v(y)$ is required to close the equation. The 1DS method prescribes a linear profile for the normal velocity that varies from a value of zero at the wall to the primary grid value at the top of the subgrid. The applicability of this assumption is supported by Figure 3.2, which shows the normal velocity profile obtained from a zero-pressure gradient turbulent boundary layer simulation with a low-Re eddy-viscosity model and a well resolved near-wall mesh. The resulting 1-D momentum equation to be solved on the subgrid is:

$$-\rho \frac{u V}{Y} + \rho \frac{V}{Y} \frac{du}{dy} = -\frac{\partial P}{\partial x} + \frac{d}{dy} \left( \mu \frac{du}{dy} \right) - \frac{d (\rho u'v')}{dy}. \tag{3.2}$$
The subgrid solution is coupled to the primary grid momentum equation through the wall shear stress, which is computed based on the velocity gradient at the wall:

\[ \tau_w = \mu \frac{du}{dy} \bigg|_{y=0}. \quad (3.3) \]

### 3.1.2 Turbulence Equations

Turbulence model equations are solved simultaneously on the subgrid. These equations are made one-dimensional by assuming local equilibrium, so that the convective terms are set equal to zero. The results presented in this paper were obtained using the one-equation models of Wolfshtein [11] and Spallart-Allmaras [12], which require solution of the general turbulence model equation on the primary grid:

\[ \rho U_j \frac{\partial \phi}{\partial x_j} = P - D + \frac{\partial}{\partial x_j} \left[ \Gamma \frac{\partial \phi}{\partial x_j} \right], \quad (3.4) \]

where \( \phi \) is the modeled turbulence quantity. The turbulent shear stress required in Eq. (3.1) is given by:

\[ \rho \overline{U_i'U_j'} = \frac{2}{3} K \delta_{ij} - 2 \mu_t S_{ij}. \]
Using the local equilibrium assumption, the corresponding 1-D equation solved on the subgrid is:

\[
0 = P - D + \frac{\partial}{\partial y} \left[ \Gamma \frac{\partial \phi}{\partial y} \right].
\]  

(3.5)

This subgrid solution is used to provide the turbulence production in the first layer primary grid cell. The diffusion coefficient, turbulent production, and turbulent dissipation rate are prescribed by the particular turbulence model and are treated in Chapter IV.
3.1.2.a The Wolfshtein One-Equation Model

With the Wolfshtein model for turbulent kinetic energy, Eq. (3.4) is rewritten as:

\[
\rho U_j \frac{\partial \phi}{\partial x} = P_k - \rho \varepsilon + \frac{\partial}{\partial y} \left( \left( \mu + \frac{\mu_T}{\sigma} \right) \frac{\partial K}{\partial y} \right).
\]

The turbulent viscosity, turbulent production, and turbulent dissipation rate are defined as:

\[
\mu_T = \rho C_m \sqrt{\overline{K} l_\mu},
\]

\[
P_k = \mu_T S^2,
\]

\[
\varepsilon = \frac{K^{3/2}}{l_D}.
\]

The length scales used in the turbulent viscosity and dissipation rate are algebraically prescribed in terms of the wall distance and wall Reynolds number:

\[
l_\mu = \left[ 1 - \exp \left( - \frac{Re_y}{A_\mu} \right) \right] C_L y
\]

\[
l_D = \left[ 1 - \exp \left( - \frac{Re_y}{A_D} \right) \right] C_L y
\]

where

\[
Re_y = \frac{\rho \sqrt{K} y}{\mu}.
\]

The analogous 1-D equation solved on the subgrid is:
\[ 0 = P_k - \rho \varepsilon + \frac{d}{dy} \left[ \left( \mu + \frac{\mu_T}{\sigma} \right) \frac{dk}{dy} \right], \quad (3.7) \]

and the turbulence model quantities are determined based on computed subgrid variables.

Finally, the turbulence production required for the first layer primary grid cell is:

\[ P_k = \mu_T \left( \frac{d u}{d y} \bigg|_{y=Y} \right)^2. \quad (3.8) \]

3.1.2.b The Spalart-Allmaras One-Equation Model

The model of Spalart and Allmaras [12] requires solution of the turbulent kinematic viscosity equation on the primary grid:

\[ \rho \frac{\partial \tilde{\rho}}{\partial x_i} = G_v + \frac{1}{\sigma_v} \left[ \frac{\partial}{\partial x_j} \left( \mu + \rho \tilde{v} \frac{\partial \tilde{v}}{\partial x_j} \right) + C_{b_2} \rho \frac{\partial \tilde{v}}{\partial x_j} \frac{\partial \tilde{v}}{\partial x_j} \right] - Y_v, \quad (3.9) \]

where the turbulent viscosity, turbulent production, and turbulent destruction rate are given by:

\[ \rho U_i \partial \tilde{v} = -2 \mu_T S_{ij}, \]

\[ \mu_T = \rho \tilde{v} f_{vl}, \]

\[ G_v = C_{b_1} (1 - f_{_{\text{el}}}) \rho \tilde{S}_v \tilde{v}, \]

\[ Y_v = \left( C_{w_1} f_w - \frac{C_{b_2}}{K} f_{_{\text{el}}} \right) \rho \left( \frac{\tilde{v}}{\sqrt{y}} \right)^2. \]

The viscous damping functions used in the turbulent viscosity and dissipation rate are algebraically prescribed in terms of the kinematic viscosity ratio:
\[ f_{vl} = \frac{X^3}{X^3 + C_{vl}^3}, \]

\[ f_{t_{2}} = C_{t_{2}} e^{-C_{u}x^2}, \]

\[ f_{w} = g \left[ \frac{1 + C_{w1}^6}{g^6 + C_{w2}^6} \right]^{1/6}, \]

where

\[ g = r + C_{w2} |r^6 - r|. \]

The analogous 1-D equation solved on the subgrid is:

\[ 0 = G_v + \frac{1}{\sigma_v} \frac{d}{dy} \left[ (\mu + \rho \tilde{v}) \frac{d \tilde{v}}{dy} \right] - Y_v + \frac{C_{\rho}^2}{\sigma_v} \rho \left( \frac{\partial \tilde{v}}{\partial y} \right)^2, \]  (3.10)

and the turbulence model quantities are determined based on computed subgrid variables.

The subgrid solution is also used to provide the turbulence production in the first layer primary grid cell:

\[ G_v = C_{bl} (1 - f_{t_{2}}) \rho \left( S_{v, subgrid} + \frac{\tilde{v}}{\kappa^2 y^2} f_{v_{2}} \right) \tilde{v}, \]  (3.11)

Similarly, the gradient based production term in Eq. (3.9) is calculated in the near-wall primary grid cells using the gradient \( \frac{\partial \tilde{v}}{\partial y} \) obtained from the subgrid solution.

### 3.2 Implementation

The subgrid illustrated in Figure 3.1 can be constructed \textit{a priori} for any simulation in order to satisfy selected mesh constraints. The topmost subgrid boundary
must correspond to the wall distance of the primary cell node. In this study, the first subgrid node is specified at a wall distance corresponding to $y^+ \approx 1$. As illustrated in Figure 3.1, the distribution of subgrid cells can be defined using a constant geometric stretching ratio, where stretching ratio of 1 yields a uniform subgrid. The value of the stretching ratio therefore determines the number of subgrid nodes within each primary grid cell. However, care must be taken in the selection of the stretching ratio to avoid an adverse impact on solution accuracy.

A subgrid refinement study is useful in appropriate selection of this ratio. Skin friction profiles, number of subgrid cells and convergence times for such a study are presented in Figure 3.3 and Table 3.1. These data are obtained for a zero pressure gradient turbulent boundary layer simulation with a coarse ($y^+ >> 1$) near-wall mesh. The results indicate that a stretching ratio of 1.2 produces a dramatic reduction in the number of subgrid cells with little loss in solution accuracy. Based on this conclusion, a subgrid stretching ratio of 1.2 is used for all of the test cases presented in this thesis.
Table 3.1: Subgrid nodes and convergence times for various stretching ratios. Values are obtained for the same flow as in Figure 3.3.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>No. Subgrid Nodes</th>
<th>Iterations to Convergence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>513</td>
<td>5400</td>
</tr>
<tr>
<td>1.2</td>
<td>27</td>
<td>825</td>
</tr>
<tr>
<td>1.5</td>
<td>15</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>800</td>
</tr>
</tbody>
</table>
CHAPTER IV

IMPLEMENTATION WITHIN THE FLUENT CFD SOLVER

The One-Dimensional Subgrid (1DS) method was coded in the C programming language and incorporated within the Fluent Computational Fluid Dynamics (CFD) framework by way of the User-Defined Function capability available with that solver. This implementation allows coding to focus on the 1DS method itself as issues such as data structure and solver integration are handled internal to Fluent.

The one-dimensional subgrid equations (Eq. (3.2) and (3.5)) are solved within each near-wall primary grid cell during each iteration of the primary grid solution. To further accelerate convergence, the subgrid velocity and turbulent kinetic energy are normalized by the primary grid values, and the normalized equations are solved on the subgrid. The use of normalized values ensures that successive subgrid solutions are continuous as the subgrid variables always lie between zero at the wall and unity at the primary grid node, regardless of changes in primary grid values from one iteration to the next.

The 1-D subgrid equations form a tri-diagonal system of equations. This system is solved during each iteration using a tri-diagonal matrix algorithm (TDMA), whereby a simplified form of Gaussian elimination is used in successive sweeps to solve the system.
The 1st sweep eliminates terms on the lower diagonal, thus resulting in a reduced set of equations. The 2nd sweep back substitutes this reduced system to produce the solution. Eqs. (3.2) and (3.5) are coupled, non-linear equations, so full convergence requires iterative application of the TDMA. As implemented, one complete pass of the TDMA is performed during each outer iteration of the primary grid solution, and the subgrid equations converge concurrently with the primary grid equations during the course of the simulation.

4.1 Momentum Equations

4.1.1 Primary Grid

Solution of the momentum equations on the primary grid is handled internal to Fluent, the user need only supply the appropriate boundary conditions. These equations take the form:

$$\frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ (\mu + \mu_t) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]$$

The 1DS method prescribes the wall boundary condition for the momentum equations using values computed on the subgrid. Thus, the wall boundary condition for the primary grid should be set as defined by Eqn. (3.3). This is accomplished in Fluent by specifying a user-defined profile for wall shear stress as the wall boundary condition. For stability, a linearized version of Eqn. (3.3) is also included as a source term for the solution within the wall adjacent cells. The linearized equation used here is:
\[ \tau_w = \left( \mu \frac{U}{y} \frac{du}{dy} \right) \bigg|_{y=0} \]

### 4.1.2 Subgrid

Solution of the subgrid momentum equation (Eqn. (3.2)) consists of the following implicit system for streamwise velocity, \( u \):

\[ a \cdot u = b \]

where \( a \) is the coefficient matrix and \( b \) is the solution vector. Determination of these coefficients at each subgrid node and subsequent solution using a tridiagonal matrix algorithm is necessary to complete the solution. For the general subgrid structure shown in Figure 4.1, the coefficient matrix for a given point \( i \) is:

\[ a_i = - \left( \frac{\mu_{x,n}}{\Delta x_n dp_n} + \frac{\mu_{x,s}}{\Delta x_s dp_s} \right) \]

and the solution vector is:

\[ b_i = \frac{dp}{dx} \left( \frac{1}{U} \right) - \rho \frac{V}{Y} u_i + \rho y_i \frac{V}{Y} \left( \frac{u_n - u_s}{2 dp_i} \right) \]

Iterative solution of this system using a tridiagonal matrix algorithm ensures that the subgrid solution converges concurrently with the primary grid solution.
Fluent is a robust CFD solver which contains its own implementations of turbulence models and their corresponding near-wall treatments. The flow equations can be activated or deactivated independently of each other during the simulation; which allows for flexibility in implementing turbulence models which are not natively included with the solver. However, there is no obvious way to completely disable the internal near-wall treatments. For this reason, the turbulence equations are deactivated in Fluent and handled entirely using the User-Defined Scalar (UDS) equation solver which allows for the solution of any number of equations having the general form:

\[
\frac{\partial}{\partial x_i} \left( \rho u_i \phi_k - \Gamma_k \frac{\partial \phi_k}{\partial x_i} \right) = S_{\phi k}, \tag{4.1}
\]
where $\phi_k$ is the solution variable, $\Gamma$ is a diffusion coefficient and $S_{\phi_k}$ is a source term. This equation is solved at each iteration of the CFD solver for user quantities of the diffusion coefficient and source term. This data type is beneficial in that the solution history is tracked in the form of residuals and gradients, just as is done for the flow momentum and turbulence equations within the main CFD solver. By using UDS equations to represent turbulence effects, there is no question whether or not Fluent's internal near-wall treatments affect the results from the 1DS method.

The source term and diffusion coefficient required for Eqn. 4.1 are taken directly from the particular turbulence model implemented. For the Wolfshtein model, these quantities can be inferred directly from Eqn. 3.6 as:

$$S_{\phi_k} = \mu, S^2 - \rho \epsilon$$

and

$$\Gamma = \mu + \frac{\mu_l}{\sigma}.$$ 

This source term is passed into Fluent's UDS solver, along with a linearized source term to promote a stable solution. This linearized source is determined by taking the derivative of the source term with respect to the variable $K$:

$$S_{\phi_k} = 1.5 \rho \frac{\sqrt{K}}{l_p}.$$ 

For the Spalart-Allmaras turbulence model, the UDS source term and diffusion coefficient, taken from Eqn. 3.9, are:

$$S_{\phi_k} = G_v + \frac{1}{\sigma_v} C_{b2} \rho \frac{\partial \tilde{v}}{\partial x_j} \frac{\partial \tilde{v}}{\partial x_j} Y_v.$$
and

\[ \Gamma = \mu + \rho \tilde{\nu} . \]

The source term is applied in the same way as is the term for the Wolfshtein model, so that the appropriate linearization for the Spalart-Allmaras model is derived by differentiating the source term with respect to the variable \( \tilde{\nu} \):

\[ S_{\phi k} = -C_{bi} f_{i2} S_{\nu} \rho S_{\nu} - 2C_{wi} f_{w} \rho \frac{\tilde{\nu}}{Y^2} . \]

### 4.2.2 Subgrid

The subgrid solution for turbulent kinetic energy is approached in the same manner as the solution for streamwise velocity in that we wish to solve the matrix equation:

\[ a \cdot \phi = b , \]

for the turbulence variable \( \phi \). The quantities \( a \) and \( b \) for the Wolfshtein model for turbulent kinetic energy, \( k \), are:

\[ a_i = -\left( \frac{\mu_{e,n}}{\Delta x_n} + \frac{\mu_{e,s}}{\Delta x_s} \right) - \rho \frac{\sqrt{k_i}}{l_d} \]

and

\[ b_i = -\mu_i \left( \frac{u_n - u_s}{y_n - y_s} \right)^2 . \]

Notice that the turbulent destruction term from Eqn. 3.7 is included in the coefficient matrix, \( a \), as opposed to the solution vector \( b \). Exclusion of non-negative quantities from the solution vector ensures maximum stability of the subgrid solution[1]. Consequently, these quantities must be moved to the other side of the matrix equation as shown here.
The coefficient matrix and solution vector for the Spalart-Allmaras model are:

\[ a_i = - \frac{1}{\sigma_v} \left( \frac{\mu_{e,n}}{\Delta x_n} + \frac{\mu_{e,s}}{\Delta x_s} \right) - \left( C_{w1} f_w - \frac{C_{b2}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{v}_i}{y^2} \right) \]

and

\[ b_i = - C_{h1} (1 - f_{t2}) \rho \tilde{S}_v \tilde{v}_i - C_{b2} \rho \left( \frac{\Delta \tilde{v}_i}{\Delta y_i} \right)^2. \]
CHAPTER V
RESULTS: BOUNDARY LAYER FLOW

In this section, the One-Dimensional Subgrid (1DS) method is applied to several types of boundary layer flows. For comparison, the low-Reynolds number (LR) and standard wall function (WF) methods (see sections 2.1 and 2.2) are also applied to these cases. The turbulence closure of Wolfshtein for turbulent kinetic energy is used to obtain the results presented in this chapter.

Each of these methods are tested on five different primary grids with differing near-wall resolution in order to investigate the sensitivity of each to grid refinement. These grids utilize a structured near-wall mesh and differ only in their nodal density and 1\textsuperscript{st} node spacing in a fixed region near the wall. Table 5.1 lists node spacing and relative coarseness for each grid in this region. In addition, the LR method is implemented with a sixth, and finest grid as a reference solution (Grid 6). The first node for this reference grid is located at $y^+ \approx 1$. By comparison, the coarsest mesh (Grid 1) has the first near-wall primary grid node at a distance 512 times greater than the reference mesh. For the 1DS method, the stretching ratio $\gamma = 1.2$ is used on the subgrid.
Table 5.1: Near-wall mesh spacing and relative grid coarseness.

<table>
<thead>
<tr>
<th>No. Nodes</th>
<th>Grid1</th>
<th>Grid2</th>
<th>Grid3</th>
<th>Grid4</th>
<th>Grid5</th>
<th>Grid6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} Wall Spacing (m)</td>
<td>2.00E-002</td>
<td>1.00E-002</td>
<td>2.50E-003</td>
<td>6.24E-004</td>
<td>1.50E-004</td>
<td>3.91E-005</td>
</tr>
<tr>
<td>Relative Coarseness</td>
<td>512</td>
<td>256</td>
<td>64</td>
<td>16</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

5.1 Zero Pressure Gradient Boundary Layer

Flow over a 2-D, zero pressure gradient (ZPG) flat plate, with a Reynolds number of of $8.2 \times 10^6$ based on plate length and inlet velocity, is used as a canonical test case for the 1DS method. The results presented include coefficient of friction profiles along the length of the plate, as well as near-wall profiles of streamwise velocity and turbulent kinetic energy at selected locations along the plate.

5.1.1 Coefficient of Friction

Calculations of skin friction coefficient are sensitive to near-wall resolution since the shear stress is proportional to the velocity gradient at the wall. Neglect of the linear sublayer due to improper 1\textsuperscript{st} primary node placement can lead to significant error in predicted values. The LR method is therefore expected to suffer as the 1\textsuperscript{st} grid node moves out of the viscous sublayer. Similarly, the accuracy of the WF method is dependent on the 1\textsuperscript{st} primary grid node being located within the logarithmic region of the turbulent boundary layer. Figures 5.1 – 5.3 show skin friction coefficient profiles calculated on the five grids for each method investigated (LR, WF, 1DS). As expected,
both the LR and WF methods are sensitive to the location of the 1\textsuperscript{st} grid node. The sudden jump in Figure 5.2 for Grid 5 is due to switching between logarithmic and linear profiles within the wall function as $y^+$ moves across the predefined threshold. In contrast to these two approaches, the 1DS method is relatively insensitive to wall refinement, as shown in Figure 5.3. For even the coarsest grid, the results approach the reference result as $Re$ increases. It should be pointed out that all of the methods tend to under predict the skin friction coefficient near the plate leading edge as the mesh is coarsened. This is because the boundary layer height is actually smaller than the near-wall cells at these locations, and so cannot be accurately resolved by any of the near wall treatments. Both the WF and 1DS methods, however, show improved prediction farther downstream as the boundary layer thickens and is able to be resolved by the primary mesh.

![Low Reynolds Number Method](image)

Figure 5.1: Skin friction coefficient distribution for ZPG flow using the LR formulation.
Figure 5.2: Skin friction coefficient distribution for ZPG flow using the WF formulation.

Figure 5.3: Skin friction coefficient distribution for ZPG flow using the 1DS formulation.
5.1.2 Velocity Profiles

Predictions of streamwise velocity at Reynolds numbers of $1 \times 10^6$ and $4 \times 10^6$ are plotted in Figures 5.4 – 5.6 for each method. For brevity, only Grids 1, 3 and 5 are considered in the remaining sections. Here the source of error in Figures 5.1 and 5.2 is apparent, as inability to reproduce the correct velocity profile translates directly into inability to correctly predict the skin friction coefficient. It is evident from Figure 5.4 that the LR method is wholly unable to produce the correct profile as the 1st primary node is moved outside the viscous sublayer ($y^+ > 5$). This error is expected since the variation of $u^+$ shifts from linear in the viscous sublayer to logarithmic outside the buffer region.

The WF method performs well for coarser grids, but yields inaccurate predictions as the 1st primary node approaches the buffer layer. The WF method is inappropriate for application in the viscous sublayer as shown in Figure 5.2 above, and so data for Grid 5 has been omitted from Figure 5.5.

Data from the subgrid solution has been added to Figure 5.6 so that the profiles are continuous down to the wall. The predicted profiles for the 1DS method are relatively invariant regardless of grid refinement and fit the reference curve closely for both Reynolds numbers presented. The worst agreement is seen on the coarsest grid (Grid 1) for Re = $1 \times 10^6$, due to the slight over prediction of wall shear stress at this location. Overall, these results suggest that the assumptions used to derive the one-dimensional governing equations are appropriate, at least for the zero-pressure-gradient boundary layer, and highlight the flexibility of the new method with regard to near-wall cell sizing.
Figure 5.4: Dimensionless velocity profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the LR formulation.

Figure 5.5: Dimensionless velocity profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the WF formulation.
Profiles of turbulent kinetic energy at Re = 1\times10^6 and 4\times10^6 are shown in Figures 5.7 – 5.9. As with the wall shear stress and velocity profiles above, the LR method is unable to predict the correct turbulent kinetic energy profile for the coarsest two grids. The Grid 5 solution, with the first primary grid node located at y^+ = 2, reproduces the reference solution with reasonable accuracy.

The results for WF shown in Figure 5.8 are closer to the reference solution than those of the LR method for Grids 1 and 3. Even so, these solutions still suffer from error.
in determining the turbulent kinetic energy peak. Again, the WF Grid 5 solution is omitted for the reasons cited above.

For the 1DS results shown in Figure 5.9, subgrid data has been included to extend the profile to the wall as in Figure 5.6. The $k^+$ profiles for the 1DS method follow the reference solution closely with the exception of Grid 3. The jump seen in the Grid 3 solution is likely due to the location of the 1st primary grid node within the buffer layer. It is apparent from Figures 5.6 and 5.3, however, that this discontinuity has only a minimal impact on the mean velocity and skin friction profiles, and still yields a result that is superior to both the LR and WF methods for this grid resolution.

![Low Reynolds Number Method](image)

Figure 5.7: Dimensionless turbulent kinetic energy profiles for ZPG flow at Re = $1 \times 10^6$ (left) and Re = $4 \times 10^6$ (right), using the LR formulation.
Figure 5.8: Dimensionless turbulent kinetic energy profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the WF formulation.

Figure 5.9: Dimensionless turbulent kinetic energy profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the 1DS formulation.
Computational costs associated with each of the methods on Grids 1, 3 and 5 are presented in Table 5.2. Computational times were determined using a single processor SunBlade 1500 workstation. As expected, the WF and 1DS methods offer significant savings in computational effort relative to the LR approach. The new method is nearly equivalent to wall functions in terms of overall computational cost on coarse meshes, and nearly equivalent to the low-Re method on fine meshes.

Table 5.2: Computing times for various near-wall grid resolutions.

<table>
<thead>
<tr>
<th></th>
<th>Lvl1</th>
<th>Lvl3</th>
<th>Lvl5</th>
<th>Lvl6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LR</td>
<td>1DS</td>
<td>WF</td>
<td>LR</td>
</tr>
<tr>
<td>No. of Iterations</td>
<td>675</td>
<td>900</td>
<td>875</td>
<td>775</td>
</tr>
<tr>
<td>CPU Time / Iteration (s)</td>
<td>0.74</td>
<td>0.8</td>
<td>0.74</td>
<td>1.07</td>
</tr>
<tr>
<td>Total CPU Time (s)</td>
<td>502.2</td>
<td>719.1</td>
<td>649.25</td>
<td>828.48</td>
</tr>
<tr>
<td>Relative CPU Time</td>
<td>0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.04</td>
</tr>
</tbody>
</table>

5.2 Favorable Pressure Gradient Boundary Layer

Flow over a 2-D, flat plate with a favorable pressure gradient (FPG), and a Reynolds number of $6 \times 10^6$ based on plate length and inlet velocity, is used to evaluate the 1DS method with respect to favorable pressure gradients. This pressure gradient is created by modeling the flow through a channel with a converging section as shown in Figure 5.10. The converging section begins at Reynolds number of $2 \times 10^6$ and acts to accelerate the flow along the length of the plate, thereby compressing the boundary layer and increasing the shear stress acting on the plate surface. The acceleration parameter
along the plate is shown in Figure 5.11, with a maximum value of $K = 2.5 \times 10^{-7}$ in the accelerating region. Results presented here are plots of coefficient of friction along the length of the plate for grids 1, 3 and 5 as compared to the reference, Grid 6.

Figure 5.10: Geometry for FPG flat plate modeled as converging channel flow.

Figure 5.11: Acceleration parameter for the FPG flat plate. $K$ reaches a maximum in the accelerating section.
Results for this test case are similar to those of the zero pressure gradient in that the LR method is unable to correctly predict $C_f$ profiles for all but the most refined grids, while the WF and 1DS methods offer marked improvement due to their treatment of near-wall effects as shown in Figures 5.12 - 5.14. The addition of the favorable pressure gradient does not seem to explicitly affect the WF method as the results agree with the reference solution downstream of the accelerating region. However, subsequent compression of the boundary layer causes the results for Grid 3 to undershoot the reference slightly as the $y^+$ value moves away from the logarithmic region. For Grid 5, the WF solution defaults to the LR method because the 1st grid node has moved below the wall function cutoff threshold.

Figure 5.12: Skin friction coefficient distribution for FPG flow obtained using the LR formulation.
Figure 5.13: Skin friction coefficient distribution for FPG flow obtained using the WF formulation.

Figure 5.14: Skin friction coefficient distribution for FPG flow obtained using the 1DS formulation.
Similarly, the 1DS method does not seem affected by the addition of the pressure gradient. Also, the method is not affected by the boundary layer compression as the results from grid attain the reference solution downstream. Thus, the 1DS method is able to handle this favorable pressure gradient while remaining relatively insensitive to the location of the 1st near-wall node within the boundary layer.

5.3 Adverse Pressure Gradient Boundary Layer

The flow over a 2-D, flat plate with an adverse pressure gradient (APG), and a Reynolds number of of $6 \times 10^6$ based on plate length and inlet velocity, is used to evaluate the 1DS method with respect to adverse pressure gradients. The pressure gradient is imposed in a similar manner as is the favorable pressure gradient of the previous section, except that a diverging section is used instead of a converging section which begins at a Reynolds number of $2 \times 10^6$. The acceleration parameter reaches a minimum of $K = -1.7 \times 10^{-7}$ in the diverging section. For this case, the pressure gradient decelerates the flow and the boundary layer is enlarged as the mean flow slows in the decelerating region.

Profiles of coefficient of friction along the length of the plate are presented in Figures 5.15 – 5.17. The same trends that are present in the previous two sections are seen here in that the LR method does a poor job in predicting the correct profile of coefficient of friction along the plate, while the WF method offers improvement over the LR results but still exhibits sensitivity to near-wall placement of the 1st primary node. Results for Grid 3 in Figure 5.16 under predict the reference solution downstream where
the results are expected to agree. The 1DS method, however, does matches the reference solution for all Grids as shown in Figure 5.17.

![Graph of Skin friction coefficient distribution for APG flow obtained using the LR formulation.](image)

Figure 5.15: Skin friction coefficient distribution for APG flow obtained using the LR formulation.
Figure 5.16: Skin friction coefficient distribution for APG flow obtained using the WF formulation.

Figure 5.17: Skin friction coefficient distribution for APG flow obtained using the 1DS formulation.
5.4 Adverse Pressure Gradient Boundary Layer With Separation

The flow over a 2-D, flat plate from the previous section is modified to produce a minimum acceleration parameter \( K = -8.4 \times 10^{-7} \) in the diverging section to evaluate the 1DS method for separating flow. The results presented in this section include coefficient of friction along the length of the plate.

Profiles of coefficient of friction along the plate for the three methods are shown in Figures 5.18 – 5.20. Not only is the LR method unable to correctly predict the profile for \( C_f \) for the coarser grids where the 1\textsuperscript{st} node spacing is well outside the viscous sublayer, but separation, which occurs as \( C_f \) passes through zero, is also not predicted for these grids. The WF method improves upon the LR results but again, shows sensitivity to the 1\textsuperscript{st} node location within the boundary layer, particularly in the region near separation. For grids 1 and 3, the predicted profiles become irregular just before separation, this is evidence that the WF method is ill suited for separating flows.

In contrast, the results for the 1DS method all converge to the reference solution downstream of the decelerating region and show none of the irregularities seen in the WF method. Grids 1 and 3 deviate from the reference only slightly just before separation; practically matching the reference solution after reattachment. Furthermore, the location of separation for this method is much closer than that predicted by the WF method. Figure 5.21 is a comparison between the WF and 1DS methods in the region of separation. Here the irregularities mentioned for the WF method are easily seen and contrast sharply with the results of the 1DS method.
Figure 5.18: Skin friction coefficient distribution for APG, separated flow obtained using the LR formulation.

Figure 5.19: Skin friction coefficient distribution for APG, separated flow obtained using the WF formulation.
Figure 5.20: Skin friction coefficient distribution for APG, separated flow obtained using the 1DS formulation.

Figure 5.21: The WF and 1DS methods in the region of separation for APG, separated flow.
CHAPTER VI
RESULTS: ALTERNATIVE IMPLEMENTATIONS

In the previous section, the One-Dimensional Subgrid (1DS) method is validated using the turbulence closure of Wolfshtein and structured near-wall topology for boundary layer flow with various pressure gradients. This section investigates other implementations of the 1DS method which take advantage of alternative turbulence closures and near-wall topologies. The general formulation of this method assumes nothing of the turbulence equations used nor the type of cells in which the subgrid is applied, thus, these new implementations are straight forward.

6.1 The Spalart-Allmaras Turbulence Closure

The zero pressure gradient (ZPG) boundary layer is revisited using the turbulence closure of Spalart and Allmaras [12] for turbulent viscosity. This implementation uses the same grids and flow conditions as the case investigated in section 5.1 since only the primary grid and subgrid turbulence equations are changed. The 1DS method is compared to the Low-Reynolds Number (LR) and Wall Function (WF) methods in terms of coefficient of friction, streamwise velocity profiles and profiles of viscosity ratio. These results are compared to a reference LR solution computed on a grid in which the 1st
grid node is located at $y' \approx 1$. For convenience, the node spacings and relative coarseness for the grids used in this section are repeated in Table 6.1.

Table 6.1: Near-wall mesh spacing and relative grid coarseness.

<table>
<thead>
<tr>
<th>No. Nodes</th>
<th>Grid1</th>
<th>Grid2</th>
<th>Grid3</th>
<th>Grid4</th>
<th>Grid5</th>
<th>Grid6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Wall Spacing (m)</td>
<td>2.00E-002</td>
<td>1.00E-002</td>
<td>2.50E-003</td>
<td>6.24E-004</td>
<td>1.50E-004</td>
<td>3.91E-005</td>
</tr>
<tr>
<td>Relative Coarseness</td>
<td>512</td>
<td>256</td>
<td>64</td>
<td>16</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

6.1.1 Coefficient of Friction

Profiles for coefficient of friction along the plate are presented in Figures 6.1 – 6.3. As seen with the results of section 5.1, the LR method performs poorly for near-wall mesh sizings that place the 1st grid node outside of the viscous sublayer. While the WF method offers notable improvement over the LR method for these grids, it noticeably under predicts the coefficient of friction on Grid 4 which has a refined near-wall mesh. This is due to the WF method's sensitivity to near-wall mesh sizing, as the method is not applicable below $y' \approx 11.5$. For mesh sizings whose 1st grid node lies below this threshold, the WF method reverts to the LR solution. The 1DS method exhibits further improvement over the WF method in that predicted results agree well with the reference solution regardless of the near-wall mesh sizing. As with the results from the Wolfshtein model, none of the methods are able to correctly predict the coefficient of friction near
Figure 6.1: Skin friction coefficient distribution for ZPG flow and the LR formulation.

Figure 6.2: Skin friction coefficient distribution for ZPG flow and the WF formulation.
the leading edge of the plate on the coarser grids since the boundary layer height is smaller than the cells in this region.

Figure 6.3: Skin friction coefficient distribution for ZPG flow and the 1DS formulation.

6.1.2 Velocity Profiles

Streamwise velocity profiles at Reynolds numbers of $1 \times 10^6$ and $4 \times 10^6$ are plotted in Figures 6.4 – 6.6 for each method. Grids 2 and 4 have been omitted for brevity. With the coarser grids, the 1st grid node lies outside of the viscous sublayer and accurate prediction of the velocity profile is impossible without proper near-wall treatment. This effect is seen in the results for the LR method, as improper prediction of the velocity profile directly leads to the errors in the coefficient of friction results of the previous
Results for the WF method are dramatically better than those of the LR method since the non-linear variation of streamwise velocity with wall distance is accounted for with this method. Even so, the WF method exhibits some mesh sensitivity. In particular, the profile in Figure 6.5 for Grid 1 at $Re = 1 \times 10^6$ is under predicted while the profile for the same grid at $Re = 4 \times 10^6$ almost exactly matches the reference solution. The 1DS method further improves these predictions and matches the reference solution for all grids except Grid 1 at $Re = 1 \times 10^6$. Despite this discrepancy, the values extracted from the subgrid solution match the reference solution in all cases. Thus, the poor prediction for Grid 1 at $Re = 1 \times 10^6$ may be a consequence of the Spalart-Allmaras model itself since both the WF and 1DS methods exhibit this behavior.

Figure 6.4: Dimensionless velocity profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the LR formulation.
Figure 6.5: Dimensionless velocity profiles for ZPG flow at \( \text{Re} = 1 \times 10^6 \) (left) and \( \text{Re} = 4 \times 10^6 \) (right), using the WF formulation.

Figure 6.6: Dimensionless velocity profiles for ZPG flow at \( \text{Re} = 1 \times 10^6 \) (left) and \( \text{Re} = 4 \times 10^6 \) (right), using the 1DS formulation.
6.1.3 Turbulent Viscosity Profiles

Figures 6.7 – 6.9 show profile of the kinematic viscosity ratio predicted with the Spalart-Allmaras model. As in the previous sections, the LR method cannot predict the correct profile for the coarser grids. Similarly, the WF method offers improvement yet still suffers from mesh sensitivity. Furthermore, this sensitivity is inconsistent for the two Reynolds numbers examined in Figure 6.8, with Grid 3 performing better for Re = 1 ×10^6 and Grid 1 performing better for Re = 4×10^6. The 1DS method, however, is relatively insensitive to the near-wall mesh with results as good as, if not better than those produced with the WF method. The worst agreement in Figure 6.9 occurs with Grid 1, though the subgrid profile quickly acquires the reference solution as the wall is approached.

![Low Reynolds Number Method](image)

Figure 6.7: Dimensionless kinematic viscosity profiles for ZPG flow at Re = 1×10^6 (left) and Re = 4×10^6 (right), using the LR formulation.
Figure 6.8: Dimensionless kinematic viscosity profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the WF formulation.

Figure 6.9: Dimensionless kinematic viscosity profiles for ZPG flow at $Re = 1 \times 10^6$ (left) and $Re = 4 \times 10^6$ (right), using the 1DS formulation.
6.2 Unstructured Grid Topologies

Thus far, the results in this study have been obtained using structured near-wall meshes. These meshes are ideal for simple geometries such as the flat plate but become impractical as geometric complexity increases. Thus, to be applicable to problems of general interest, the 1DS method must tolerate any type of near-wall mesh employed.

6.2.1 Zero Pressure Gradient Boundary Layer

Due to its innate simplicity, the zero pressure gradient flat plate of Chapter V with the Wolfshtein turbulence closure is used to validate the 1DS method on an unstructured topology. The same methodology is used in this investigation in that the results from the 1DS methods are compared to those from the LR and WF methods. The only difference between the test grids used here and those for the structured version of this test case is the mesh topology in the near wall region. Table 6.1 lists node spacing and relative coarseness for each grid in this region. Since obtaining $y^+ = 1$ is computationally prohibitive with this type of near wall cell, the reference (Grid 6) solution from Section 5.1 is used as the reference solution here. As a consequence, irreconcilable topology effects have been introduced in comparing unstructured solutions to a structured reference solution.
Table 6.2: Near-wall mesh spacing and relative grid coarseness for the flat plate.

<table>
<thead>
<tr>
<th>No. Nodes</th>
<th>Grid1</th>
<th>Grid2</th>
<th>Grid3</th>
<th>Grid6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Wall Spacing (m)</td>
<td>2.00E-002</td>
<td>1.00E-002</td>
<td>2.50E-003</td>
<td>3.91E-005</td>
</tr>
<tr>
<td>Relative Coarseness</td>
<td>512</td>
<td>256</td>
<td>64</td>
<td>1</td>
</tr>
</tbody>
</table>

As already mentioned, neglect of the linear sublayer due to improper 1st primary node placement can lead to significant error in predicted values. Thus, the accuracy of the LR method is expected to suffer as the 1st grid node moves out of the viscous sublayer; while that of the WF method is dependent on the 1st primary grid node being located within the logarithmic region of the turbulent boundary layer. Skin friction coefficient profiles calculated on the three grids for each method investigated (LR, WF, 1DS) are shown in Figures 6.10 – 6.12. As expected, the LR method is sensitive to the location of the 1st grid node; producing poorer results as the grid is coarsened. Results from the WF method show little downstream deviation with respect to grid refinement.
Figure 6.10: Skin friction coefficient distribution for ZPG flow and the LR formulation.

Figure 6.11: Skin friction coefficient distribution for ZPG flow and the WF formulation.
Surprisingly, the 1DS method is not as insensitive to near-wall placement as expected. In Figure 6.12, Grid 3 lies well above the reference solution. Consequently, the first primary grid node is roughly located in the buffer layer for this grid, though that this should affect the solution does not follow. Grids 1 and 2, however, are indistinguishable above $Re = 10^5$.

![Graph showing skin friction coefficient distribution for ZPG flow and the 1DS formulation.](image)

Figure 6.12: Skin friction coefficient distribution for ZPG flow and the 1DS formulation.

As seen with the other test cases, all methods tend to under predict the skin friction coefficient near the plate leading edge as the mesh is coarsened. Again, this is because the boundary layer height is actually smaller than the near-wall cells at these locations, and so cannot be accurately resolved by any of the near wall treatments. Both
the WF and 1DS methods, however, show improved prediction farther downstream as the boundary layer thickens and is able to be resolved by the primary mesh. Also, the WF and 1DS solutions are consistently offset from the reference solution. This deviation is likely a topology effect as the reference solution uses highly stretched structured cells in order attain \( y^+ \approx 1 \). Further investigation of this effect is not possible as equivalent near-wall resolution using isotropic unstructured elements is computationally prohibitive.

### 6.2.2 Circular Cylinder

Flow over a 2-D circular cylinder at Reynolds number, based on cylinder diameter of \( 1 \times 10^6 \), is also investigated with the three methods. Symmetry of the anticipated RANS solution is used to simplify the geometry by dividing the cylinder with a symmetry line so that only the top half of the cylinder is modeled. Similar to the flat plate simulations already discussed, the turbulence closure of Wolfshtein is used. Also, a limited grid refinement study is employed to bring out the inherent differences in the methods. Table 6.3 lists the node spacings and relative grid coarseness for the grids involved. For reasons listed in the previous section, the reference grid (Grid 3) uses boundary layer type structured cells in the near-wall region.
Table 6.3: Near-wall mesh spacing and relative grid coarseness for the circular cylinder.

<table>
<thead>
<tr>
<th></th>
<th>Grid1</th>
<th>Grid2</th>
<th>Grid3</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. Nodes</td>
<td>10</td>
<td>20</td>
<td>109</td>
</tr>
<tr>
<td>1st Wall Spacing (m)</td>
<td>9.00E-003</td>
<td>4.50E-003</td>
<td>2.00E-005</td>
</tr>
<tr>
<td>Relative Coarseness</td>
<td>450</td>
<td>225</td>
<td>1</td>
</tr>
</tbody>
</table>

As discussed with regards to the flat plate solutions, profiles for skin friction coefficient along the surface of the cylinder are expected to suffer for coarse grids without proper near-wall treatment. Furthermore, separation is a key feature of this flow type and accurate methods are necessary to correctly capture the separation point. Therefore, the LR method is expected to perform poorly with regards to these metrics. The WF method should provide improvement over the LR method but may be limited by its derivation as flow over the cylinder experiences both strong adverse and favorable pressure gradients. Profiles of skin friction coefficient are plotted versus the angle θ (with θ = 0 corresponding to the front of the cylinder) in Figures 6.13 – 6.15 for the LR, WF and 1DS methods respectively. The LR method suffers significantly from lack of near-wall refinement as Grid 1 predicts no separation and Grid 2 over predicts by 25%. The WF method performs better than the LR method, though discrepancies still exist when compared to the reference solution. This implementation also exhibits a shift, this time in the θ direction, similar to that mentioned in the previous section.
Figure 6.13: Skin friction coefficient distribution for the Circular cylinder using the LR formulation.

Figure 6.14: Skin friction coefficient distribution for the Circular cylinder using the WF formulation.
The 1DS method shows further improvement over the WF solution in regions of mild pressure gradients but deviates from the reference solution as the flow accelerates over the top of the cylinder. Results on Grid 1 are significantly over predicted in the region of strong favorable pressure gradient at the top of the cylinder. That this over prediction is reduced for Grid 2, which is more refined, suggests a topological cause to this problem. Since cells on the cylinder are isotropic, spacing in the θ direction also decreases as the grid is refined. Thus, the 1DS method appears to exhibit some sensitivity to grid spacing in the streamwise direction which does not similarly manifest with the WF method. Despite this, both Grids 1 and 2 reach the same solution prior to separation; which is shifted off of the reference solution. Again, this shift is possibly due

Figure 6.15: Skin friction coefficient distribution for the Circular cylinder using the 1DS formulation.

![Skin friction coefficient distribution for the Circular cylinder using the 1DS formulation.](image)
to topological differences between the coarse grids and the reference grid in the near-wall region. Predictions of the separation point is also slightly better than those of the WF method. Predicted separation for each method discussed above is tabulated in Table 6.4.

Table 6.4: Predicted separation point for each method

<table>
<thead>
<tr>
<th>Method</th>
<th>Grid1</th>
<th>Grid2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR</td>
<td>------</td>
<td>25.56%</td>
</tr>
<tr>
<td>WF</td>
<td>11.42%</td>
<td>4.92%</td>
</tr>
<tr>
<td>1DS</td>
<td>3.65%</td>
<td>3.36%</td>
</tr>
</tbody>
</table>
CHAPTER VII
CONCLUSION

The development of a new wall treatment method for turbulent flow computational fluid dynamics (CFD) simulation, based on the solution of 1-D transport equations for momentum and turbulence on an imbedded one-dimensional near-wall subgrid is presented in this thesis. The new One Dimensional Subgrid (1DS) method is found to be comparable to wall functions (WF) in terms of computational cost, while providing the accuracy of a refined low-Re model solution (LR) for zero-pressure-gradient, favorable pressure gradient and adverse pressure gradient boundary layer flow. Skin friction coefficient, streamwise velocity profiles, and profiles of turbulence model quantities utilizing the 1DS method are found to be superior to the LR and WF methods when tested on a range of near-wall structured mesh sizes. Additionally, the 1DS method is relatively insensitive to near wall grid refinement; with the only requirement that the 1st primary node be located sufficiently close to the wall so that the boundary layer can be resolved.

The 1DS method has also been successfully implemented on two unstructured geometries: zero-pressure-gradient boundary layer flow and a 2-D circular cylinder. Results for coefficient of friction are compared to those of the low Reynolds number
(LR) and wall function (WF) methods. The 1DS method shows improvement over the traditional wall function method for most cases. For the flat plate case, the method closely reproduces the results from structured ZPG boundary layer simulation mentioned above. Similarly, predictions of the separation point for flow over a circular cylinder are improved by several percent over the alternatives (LR and WF methods) on coarse grids.

The results mentioned thus far have been produced using the one-equation turbulence closure of Wolfshtein [11] which models turbulent kinetic energy. The results for the structured zero-pressure gradient boundary layer have been reproduced using an alternative turbulence closure proposed by Spalart and Allmaras [12], which solves a transport equation for turbulent viscosity, instead of turbulent kinetic energy. These results demonstrate the flexibility of the 1DS method in that it shows potential for application with any turbulence closure.

Several issues have arisen with the unstructured implementation of the 1DS method that warrant further investigation. Firstly, the coefficient of friction is significantly over predicted for zero pressure gradient flow over the unstructured flat plate with near-wall spacings that place the first primary grid node in the buffer layer. This issue is not present in the WF implementation on the same grid, nor is it apparent on a structured implementation of the 1DS method with similar near-wall mesh sizing. Second, the 1DS method exhibits some sensitivity to mesh spacing in the streamwise direction, at least for flow in the region of strong favorable pressure gradient as shown on the unstructured cylinder. This is not an issue with the structured results since the streamwise mesh spacing is uniform across the grids used. A streamwise grid refinement study using structured grids could prove enlightening with respect to this issue. Lastly,
both the WF and 1DS methods exhibit a shift in results away from the reference solution for the unstructured flat plate and cylinder cases. These shifts are likely the result of topology differences between the test and reference grids as the reference grid employs highly skewed quadrilateral cells in the near-wall region.
REFERENCES


