DESIGN OF TEST SECTIONS FOR A HIGH ENTHALPY WIND TUNNEL

By

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This document describes the design of a supersonic and a subsonic test section for a high enthalpy wind tunnel. A streamline is tracked through a supersonic test section using the method of characteristics. The specifics of the design program and the design techniques are illustrated for the supersonic section. The section of the paper dealing with the subsonic nozzle has a greatly diverse nature. This section details the inlet and exhaust restrictions and construction elements for the entire low speed system. The system is currently being set up for testing with the subsonic section, and the supersonic will eventually follow.
DEDICATION

This paper is dedicated to my family and friends that have always supported me in everything.
ACKNOWLEDGEMENTS

My heartfelt gratitude is conveyed to all who aided in the completion of my graduate work. Having Dr. Keith Koenig as my adviser and friend during this time was a tremendous advantage, and I would like to express my endless appreciation to him. I am also extremely grateful to the Mississippi/NASA Space Grant and Bob Cook and Walter Okhuysen from the Center for Advanced Energy Conversion.
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LIST OF SYMBOLS

a  upper point in field point unit process
b  lower point in field point unit process
c  point to be found in field point unit process
b  point at beginning of reflected wave (centerline index)
cp  centerline point
e  wall point on preceding wave
fa  above feeding point
fb  below feeding point
i  incident
iw  index of wave of wp
nb  number of boxes
nc  number of characteristics
r  reflected
s  curvilinear distance along streamline
s_{ab}, s_{bc}  average slope at endpoints
w  wall
wp  wall point
$C^+, C^-$  Mach waves
$K^+, K^-$  Riemann invariants
M  Mach number
$\theta$  flow angle
$\mu$  Mach angle
$\nu$  Prandtl-Meyer angle
CHAPTER I
INTRODUCTION

A high enthalpy wind tunnel, as shown in Figure 1.1, is being constructed at Raspet Flight Research Laboratory to support research on air breathing hypersonic propulsion problems. The purpose of this wind tunnel is to provide flow conditions for the exit of ramjet and scramjet combustion chambers. In addition, it will aid in the investigation of flow diagnostic methods and internal wall materials. This facility requires subsonic and supersonic test sections. The purpose of this project is to design these test sections.

The design of the supersonic section is based on the method of characteristics for planar flow. Huntington (1951) is proof that this method has been used for many years, and many computer programs exist for its application such as in Zucrow & Hoffman (1976). In the present use, one aspect is somewhat different from previous applications and a new program is needed. The program language is also different than those used in
the past. Furthermore, certain internal flow details are obtained in a unique way. For these reasons, the development of the method of characteristics program is a major element of the present work.

The flow requirements are more lax, so the subsonic section design is based largely on empirical guidelines. Concerns with the flow uniformity, design Mach number, and a few other issues are less critical than usual. In addition, more elaborate design procedures are not appropriate because the overall project needs a test section in a short time frame.

This thesis begins with a discussion on the supersonic section design. This account will focus on the details of the actual design program. Concrete design methods and individual program details will be explained.

This subsonic discussion has a much different character than the supersonic. It addresses actual construction details as well as design methods, since this test section is the first one to be built.
CHAPTER II
SUPERSOONIC TEST SECTION DESIGN

The design of the nozzle for this supersonic test section uses the shape of a tracked streamline applying the method of characteristics. First, the fundamentals of the method of characteristics are outlined. Then, specifics concerning the design approach and the program are discussed.

Method of Characteristics Solution

The basic terminology and equations for the method of characteristics, as used here, are given in the following sections. The reader is referred to one of the many good texts available, such as Hodge and Koenig (1995), Thompson (1972), and Liepmann and Roshko (1957), for a discussion of the underlying theory and derivation of these relations. This analysis does not involve free surfaces or shocks, and therefore there are no unit processes for them.

Fluid Mechanic Relations

The key fluid mechanic relations are results of the conservation of mass, momentum, and energy. Information is brought by Mach waves from the boundaries into the flow. As shown in Equation (2.1), along any given positive Mach wave, $C^+$, the difference between the flow angle and the Prandtl-Meyer angle is a constant. Similarly, Equation (2.2) demonstrates that along any given negative Mach wave, $C^-$, the sum of the flow
angle and the Prandtl-Meyer angle is a constant. These two equations are known as the Riemann invariants.

\[ K^+ = \theta - \nu = \text{constant} \]  \hspace{1cm} (2.1)

\[ K^- = \theta + \nu = \text{constant} \]  \hspace{1cm} (2.2)

This solution can be stated in a more efficient form as shown below.

\[ K^\pm = \text{constant on } C^\pm \]  \hspace{1cm} (2.3)

Figure 2.1 Characteristic Regions

The four characteristic regions – uniform, simple\(^-\), simple\(^+\), and nonsimple – are illustrated in Figure 2.1. Unit processes are a series of fundamental computational processes that execute the method of characteristics in all of the above named regions. The C\(^+\) and C\(^-\) characteristics intersect in the flow as shown in Figure 2.2. If all of the
flow properties at points a and b are known, then the following equations apply for the field points.

\[ \theta_c = \frac{1}{2}(K_c^- + K_c^+) \]  
\[ \nu_c = \frac{1}{2}(K_c^- - K_c^+) \]

The Mach number at point c can be found from the Prandtl-Meyer relation once \( \nu \) is known. The static-to-stagnation ratios at point c are calculated using the isentropic relations. Finally, if the upstream stagnation conditions are known, the local static properties are easily found due to the flow being homentropic. To calculate the angle of the reflected characteristic, \( \theta \pm \mu \) for \( C_r^\pm \), the Mach angle, \( \mu \) must be found using the following formula.

\[ \mu = \sin^{-1}\left(\frac{1}{M}\right) \]

The contact of characteristics with a wall is shown in Figure 2.3. Figure 2.3a represents the reflection of a \( C^- \) characteristic from a lower wall. Similarly, the reflection
of a $C^+$ characteristic from an upper wall is shown in Figure 2.3b. Assuming the wall shape and Riemann invariants on each incident characteristic are known, the flow angle is equal to the wall angle since the flow at the wall is tangent to the wall. At the lower and upper walls, respectively, the Riemann invariants on the incident characteristic obey the following relationships.

\[ K_i^- = K_w^- = \theta_w + v_w \]  \hspace{1cm} (2.7)

\[ K_i^+ = K_w^+ = \theta_w + v_w \]  \hspace{1cm} (2.8)

Manipulation of these relationships allows us to find $v$ at the wall, given in the following equations.

\[ v_w = K_i^- - \theta_w \]  \hspace{1cm} (2.9)

\[ v_w = \theta_w - K_i^+ \]  \hspace{1cm} (2.10)

Once $v_w$ is known, the reflected Riemann invariants are given in Equations (2.11) and (2.12) in the most useful form.

\[ K_r^+ = 2\theta_w - K_i^- \]  \hspace{1cm} (2.11)

\[ K_r^- = 2\theta_w - K_i^+ \]  \hspace{1cm} (2.12)
Geometry of the Flow

The key to finding the coordinates involves the simple geometry of the flow. The slopes of the $C^+$ and $C^-$ at any point are

$$\frac{dy}{dx} = \tan(\theta + \mu) \quad (2.13)$$

$$\frac{dy}{dx} = \tan(\theta - \mu) \quad (2.14)$$

Assuming straight-line segments between grid points as illustrated in Figure 2.4, the slopes for the two characteristics are determined from the average slope at the endpoints,

$$s_{ac} = \tan\left(\frac{1}{2}\left((\theta - \mu)_{a} + (\theta - \mu)_{c}\right)\right) \quad (2.15)$$

$$s_{bc} = \tan\left(\frac{1}{2}\left((\theta + \mu)_{b} + (\theta + \mu)_{c}\right)\right) \quad (2.16)$$
Given that the slopes and the location of points a and b are known, the equations for the y and x coordinates at point c are as follows.

\[ y_c = y_a + s_{ac} (x_c - x_a) \]  \hspace{1cm} (2.17)

\[ x_c = \frac{y_a - y_b + s_{bc} x_b - s_{ac} x_a}{s_{bc} - s_{ac}} \]  \hspace{1cm} (2.18)

This method is applicable anywhere away from a boundary.

A similar process is used to find the wall points using the intersection of the wall and the incident characteristic. The wall shape can be expressed as

\[ y_w = y_w(x) \]  \hspace{1cm} (2.19)
The incident characteristics are approximated as straight-line segments. The equations of the lower wall and a known incident characteristic, \( C_i^- \), that passes through a known point \( a \) are given in Eq. (2.20a) and (2.20b).

\[
y_w = y_w(x_w) \tag{2.20a}
\]

\[
y_w = y_a + s_{aw}(x_w - x_a) \tag{2.20b}
\]

The equations of the upper wall and a known incident characteristic, \( C_i^+ \), that passes through a known point \( b \) are given in Eq. (2.21a) and (2.10b).

\[
y_w = y_w(x_w) \tag{2.21a}
\]

\[
y_w = y_b + s_{bw}(x_w - x_b) \tag{2.21b}
\]

These equations are used to find the coordinates of the intersections. This completes the solution for the flow properties and the location of the interior and wall points.

**MOC Nozzle Design Program**

The method of characteristics described above is implemented for a minimum length nozzle and a streamline is traced in this program for a specific geometry. The centerline is treated as a lower wall with the wall angle being zero. The nozzle wall is treated as an upper wall with a shape that allows for no reflected waves. The ratio of specific heats, the gas constant, the exit Mach number, the number of characteristics, and the \( x \) and \( y \) coordinates of the origin of the centered expansion must be known for the execution of this program.
Indexing Points and Characteristics

The given number of $C^-$ characteristics that meet the centerline and reflect to the upper wall represent the initial expansion. The intersections of the characteristics with the centerline, upper wall, and each other are each given an index. The best way to demonstrate how the centerline (cp) and wall points (wp) are indexed is through the following equations and pictures. Figure 2.5 illustrates how all of the points are indexed. The first intersection of the first wave and centerline is assigned the first index. The first subroutine, $nd$, uses the following equations to form an array, $cw$, which contains the indices of the centerline and upper wall.

\begin{equation}
wp = b + [nc - (iw - 1)] \tag{2.22}
\end{equation}

\begin{equation}
cp = e + 1 \tag{2.23}
\end{equation}

Figure 2.5 Index Identification
The output, \( wpt \), of the next loop, \( wave \), is an array of the \( C^- \) passing through each index. The intersections of the characteristics and their reflections are numbered using a scheme involving feeding points. Two other indices feed each index, such as points \( a \) and \( b \), as shown in Figure 2.2. In this program, as depicted in Figure 2.6, \( fa \) and \( fb \) denote the feeding points \( a \) and \( b \), respectively, that feed point \( c \). Two simple recursion relations listed below establish each set of feeding points and are implemented in the loops \( feed_a \) and \( feed_b \), whose outputs are \( fa \) and \( fb \), respectively.

\[
fa = c - [nc - (iw - 2)] \tag{2.24}
\]

\[
fb = c - 1 \tag{2.25}
\]

Figure 2.6 Feeding Points
Flow Properties

After all of the points have been indexed, the initial values for $\theta$ and $\nu$ are defined using the Prandtl-Meyer relations. A scheme, which starts at the beginning, $b$, continuing to the end, $e$, of each wave is used in conjunction with Eq. (2.2) to find the negative Riemann invariants for all of the points in the subroutine $Km_{ns}$, whose output is $Kmd$. Similarly, $Kpd$, whose output is $Kpd$, calculates the positive Riemann invariants for all of the points that relates to Eq. (2.1).

Once the Riemann invariants are identified, $\theta$ and $\nu$ are found separately for the centerline, interior, and wall points. In the centerline loop, $\theta$ for the centerline is set to zero and $\nu$ is $K^-$ according to Eq. (2.7) in the output $cl$. Using a method like the one used for $Km_{ns}$ and $Kpd$, Equations (2.4) and (2.5) are implemented in the interior subroutine with the output, $nt$. The values for $\theta$ and $\nu$ at the wall are found in previous subroutines and calculations. The wall loop, whose output is $wll$, places all of these values together in one string. Using the Prandtl-Meyer function, the Mach number is found corresponding to the identified values of $\nu$. Then, Eq. (2.6) is applied to find the angle $\mu$. Finally, all values of $\theta$ and $\mu$ are put into one array for easier access.

Defining Coordinates for Indices

The slopes of the characteristics for each box are found as the average slope at the feeding points. The slopes are indexed according to which index they stem from. In the $slope_{ac}$ subroutine, Eq. (2.15) is used to find the slope from point $a$ to point $c$ at all points with the output, $sac$. The slopes on the initial characteristic are defined first. Next, a scheme that marches through the interior points and centerline points is executed.
Finally, the slopes for the wall points are identified. Similarly, the slope from point \( b \) to point \( c \) is determined from Eq. (2.16) in the loop \( \text{slope}_{bc} \) with the output, \( \text{sbc} \). For this loop, all of the slopes for the interior and wall points are defined using one scheme. Since there are no points below the centerline for this geometry, \( \text{sbc} \) is set to zero for the centerline points.

Now that the slopes have been defined at all points, the \( x \) and \( y \) coordinates are established. The first point on the first characteristic is labeled in the \( \text{crd1} \) subroutine. Then in the same subroutine, the coordinates for the first characteristic are defined separately using Eq. (2.17) and (2.18) so they will all be output in one array, \( \text{xyi} \). In the next loop, \( \text{coord} \), the centerline coordinates are found using Eq. (2.20b) with \( y_w \) set to zero. The remaining coordinates for the interior and wall points are output in an array, \( \text{xy} \), with the centerline coordinates using a scheme that marches through each characteristic separately applying Eq. (2.17) and (2.18). The table in Appendix A displays the flow properties and coordinates at all of the nodes in the characteristic mesh with 7 characteristics for a design exit Mach number of 3.

**Grid Trace**

Once the \( x \) and \( y \) coordinates for each index are identified, the subroutine, \( \text{plot} \), must trace the characteristics for building the grid for a plot. To set up the grid for easier plotting in Mathcad®, the trace is one curve that goes through all of the points. Some overlapping occurs. For example, in Figure 2.7, the trace begins at point 1 and goes back along the first characteristic to point 15 then along the upper wall to point 5, and down the reflected characteristic to 4, 3, 2, and 1. It then moves forward along the centerline to
point 6 and upstream along the second wave to 2 and 15, downstream along the upper wall to 5 and 9, down the second reflected wave to 8, 7, and 6, forward to 10, upstream to 7, 3, and 15, downstream to 5, 9, and 12, down to 11 and 10, forward to 13, upstream to 11, 8, 4, and 15, downstream to 5, 9, 12, and 14, and finally down to 13. The output of this subroutine, plt, is a single array of the indices in the order in which they are traced. The corner coordinate is identified, index 15 in Figure 2.7, and a set of equations assigns the coordinates to the trace.

![Figure 2.7 Grid Trace](image)

**Initial Value Line**

The method of characteristics requires an initial value line for starting. The preceding implementation of the method of characteristics assumes that the initial value line is a vertical straight line on which $M = 1$, that is the sonic line. This line was chosen after
examination of the properties of the more complex initial value line that arises from Sauer’s method (Sauer (1947)). Sauer’s method is based on a solution to the small disturbance transonic potential flow equation. The net result is a closed form expression for the curve, at the throat, on which the vertical velocity is zero. This curve serves at the initial value line.

Applying this analysis to the present geometry yields the situation shown in Figure 2.8. Sauer’s initial value line is a nearly vertical, nearly straight curve on which the Mach number varies from 1 at the top to 1.027 at the bottom. If Sauer’s line is used, a set of $C^\pm$ waves emanating from it must be introduced into the characteristic network, which complicates the analysis. Because the initial value line from Sauer’s method is closely approximated by a sonic line that is straight and vertical, the latter is used here for the initial value line. The complexity of the initial $C^\pm$ wave system is thus avoided.

![Figure 2.8 Sauer’s Method Vs. Vertical Line](image-url)
Start of Streamline

Streamline tracking begins at a point \((x_p, y_p)\), shown in Figure 2.9, which is very slightly downstream of the boundary between the simple minus and nonsimple regions. The streamline that passes through this point is followed both upstream to \(x = 0\) and downstream to the end of the nozzle. The index of the characteristic on which the streamline begins is denoted \(n_cs\). For this program, a point with the coordinates, \(x_p\) and \(y_p\) is chosen to be just past an index point on the wave on which the streamline begins. Once that point is chosen, the \(att\) subroutine marches backwards across each wave to connect in the simple minus region until the first \(C^-\) is reached. The streamline path in the short region upstream of the \(C^-\) is approximated by a circular arc, which is a common practice in nozzle design (Zucrow and Hoffman (1976)). Here the radius and origin of the circular arc are chosen so that the arc slope is zero at \(x = 0\) and matches the streamline slope at the first wave. The output, \(xys\), is an array with the \(x\) and \(y\) coordinates of the points the streamline crosses on each wave. The \(att2\) loop defines the circular arc that is used for the shape of the streamline. The output, \(xyt\), is an array of the \(x\) and \(y\) coordinates of the points on the arc. Finally, \(xyw\) is an array that combines \(xys\) and \(xyt\).

The program then returns to \(x_p\), \(y_p\) and marches downstream through the nonsimple region. This process requires determination at each step of the grid box in which the leading point of the streamline lies. A number of subroutines are used to index the boxes, identify the current box, and then advance the streamline. The streamline eventually leaves the nonsimple region and a scheme similar to \(att\) follows it through the simple plus region.
Box Identification

For boxes to be indexed, Eq. (2.26) calculates the total number of boxes.

\[ nb = 0.5 \cdot nc^2 + 0.5 \cdot nc - 1 \]  \hspace{1cm} (2.26)

Figure 2.10 demonstrates the box indexing system. The subroutine, \textit{tribox}, identifies the boxes along the centerline in an array of the indices, \textit{tri}. This set of three-sided boxes is defined so that the streamline never encounters them. Similarly, the loop, \textit{wbox}, identifies the boxes along the upper wall in an array of the indices, \textit{wbox}. A loop, \textit{box}, is set up using the feeding points to define the corner indices of every box.
The next step in tracking the streamline is to define a point based upon the box that encases it. As shown in Figure 2.9, this must be done in order to know which characteristic region the streamline is in and which formulas apply. First, the sides of each box are extended. The output, $tan_b$, of the loop, $tana$, is an array of the tangents of the extended sides of the box. This is shown in Figure 2.11, where $\alpha$ is the counterclockwise angle from the x-axis to the tangents. A unit outward normal vector is drawn from the corners of each box by rotating this angle counterclockwise 90º as shown in Figure 2.12. The loop, $tann$, outputs the tangents of vectors drawn normal to the box in an array, $tan_n$. Also, the x and y coordinates for the corners of each box in the subroutines $xcrnr$ and $ycrnr$, respectively. The subroutines, $nx$ and $ny$, output arrays, $xn$
and $xy$, respectively, contains the x and y coordinates for the tips of each normal drawn from the corners of each box.

Next, vectors are drawn from the specified point normal to each of the sides or extended sides. Finally, the sum of the dot products of each of the matching vectors is found in the subroutine, $Dot$, and used to identify the box that encases the point. This subroutine allows for every box to be checked for the new point. If any of the dot products are negative, the point is outside the box, as shown in Figure 2.13. If all of the dot products are positive, the point is inside the box, as shown in Figure 2.14. The output of this subroutine, $dot$, is the box that encases any given point.

---

**Figure 2.11 Extended Sides of Box**
Figure 2.12 Unit Outward Normals

Figure 2.13 Point Outside Box
Figure 2.14 Point Inside Box

Figure 2.15 Intersection of Extended Boxes
Nonsimple and Simple Plus Regions

Once the box for the first point is identified, an interpolation scheme is needed to find \( \theta \). If the sides of each box are extended far enough, they intersect as shown in Figure 2.15. The output of the subroutine, \( \text{Int} \), contains the \( x \) and \( y \) coordinates for these intersections in a 2x2 matrix. The next subroutine, \( \text{phi} \), finds the angles of the extended local characteristics and the ray from the point to the intersection of the extended characteristics. Once these angles are determined, \( \theta \) is found using the linear interpolation (2.27) – (2.30). If the box that encases the point is a wall box, Eq. (2.29) is used to find \( \theta \) and Eq. (2.30) if not.

\[
\begin{align*}
K^+ &= K_C^+ + \frac{K_B^+ - K_C^+}{\phi_3 - \phi_1} \cdot (\phi_2 - \phi_1) \quad (2.27) \\
K^- &= K_D^- + \frac{K_C^- - K_D^-}{\phi_6 - \phi_4} \cdot (\phi_5 - \phi_4) \quad (2.28) \\
\theta &= \frac{1}{2} \cdot (K^+ + K^-) \quad (2.29) \\
\theta &= \theta_A + \frac{\theta_D - \theta_A}{\phi_1 - \phi_3} \cdot (\phi_2 - \phi_3) \quad (2.30)
\end{align*}
\]

Finally, \( \text{phi} \) increments to the new point as follows

\[
\begin{align*}
x_{\text{new}} &= x_{\text{current}} + \Delta x \quad (2.31) \\
y_{\text{new}} &= y_{\text{current}} + \tan(\theta) \cdot \Delta x \quad (2.32)
\end{align*}
\]

where \( \Delta x \) is a specified, fixed increment in \( x \) given by

\[
\Delta x = \text{constant} \cdot x_{y1,\text{npt}-1} \quad (2.33)
\]

where \( x_{y1,\text{npt}-1} \) is the \( x \) value at the last grid point on the centerline.
The output, \( b_{xy} \), is an array that contains the box that encases each point, the x and y coordinates, and \( \theta \) for each remaining point on the streamline.

Figure 2.16a. shows a nozzle designed for air with an exit Mach number of 3 using fifteen initial characteristics at the throat. The origin of the centered expansion is at (0,1), and the streamline begins on the sixth characteristic. This figure also illustrates the tracked streamline for the possible design of the shape of the upper wall of the nozzle. Figure 2.17 is a magnification of Figure 2.16 so that changes in the streamline through the different regions are more obvious.

Figure 2.18 is a plot of Mach number, M, as a function of position along the streamline, s. It begins on the first characteristic. The Mach number between 0 and the first characteristic could have been calculated using the same method as the simple\(^-\) region. There is a definite change in slope when the streamline passes from the simple\(^-\) region into the nonsimple region and from the nonsimple into the simple\(^+\). The first change in slope is due to the larger change in step size from the simple\(^-\) to the nonsimple region. The second more pronounced jump is mostly due to the discontinuity associated with characteristics. Both Anderson (2003) and Hodge and Koenig (1995) observe that flow properties can be irregular as characteristic directions. The nearly vertical line segment shown in the second transition is the connection between the last point in the nonsimple region and the first point in the simple\(^+\) region. Because there is a miniscule jump in the points, the plot compensates with a straight-line segment.
Figure 2.16 Streamline for Nozzle Design
Figure 2.17 Magnification

Figure 2.18 Mach Number ($M$) Vs. Position Along Streamline(s)
Indexing System

Within this program, the indexing system played a major role in the output of most of the subroutines. In Mathcad®, each subroutine outputs arrays that can be used within other loops for calling or indexing purposes. For example, in the following interior point subroutine, \( cw \) is an array that has already been established containing the indexes of the centerline and upper wall. The variable \( ip \) is set up to run from just above the centerline to just below the upper wall of each wave. The indexes \( fa \) and \( fb \) denote feeding points that have already been defined in prior subroutines. Double indexes are used to help identify the Riemann invariants that are manipulated in equations to find \( \theta \) and \( \nu \) and output them into one array for all of the interior points. For example, \( Kmd_{fa_{ip}} \) in the first execution of the built in loop is the \( K^- \) value for the above feeding point for the first point above the centerline on the second characteristic. All of the arrays in this program are indexed in a certain way for this purpose.

\[
\text{interior}(nc, cw, Kmd, Kpd, fa, fb) := \begin{align*}
\text{for } iw &\in 2..nc - 1 \\
b &\leftarrow cw_{1,iw} + 1 \\
e &\leftarrow cw_{2,iw} - 1 \\
\text{for } ip &\in b..e \\
\theta v_{1,ip} &\leftarrow \left(\frac{1}{2}\right) \left[ \frac{Kmd_{fa_{ip}}}{Kpd_{fb_{ip}}} + Kpd_{fb_{ip}} \right] \\
\theta v_{2,ip} &\leftarrow \left(\frac{1}{2}\right) \left[ \frac{Kmd_{fa_{ip}}}{Kpd_{fb_{ip}}} - Kpd_{fb_{ip}} \right]
\end{align*}
\]

In this next example, the corner indices of every box are identified. The \( tri \) array was already defined as the indexes of the three-sided boxes. Above and below feeding points are used to define the corners of all of the boxes. However the three-sided boxes do not
have a below feeding point and only have 3 corners. Therefore, double indexing was applied again to identify the corners of the three-sided boxes within this loop.

box := for i ∈ 1..nb
    box_{i,1} ← i + nc
    box_{i,5} ← box_{i,1}
    box_{i,4} ← nc + i + 1
    box_{i,3} ← fa(box_{i,4})
    box_{i,2} ← fb(box_{i,3})
    for j ∈ 1..rows(tri)
        box_{tri,1} ← 0
        box_{tri,5} ← 0
    box
CHAPTER III

SUBSONIC TEST SECTION DESIGN AND CONSTRUCTION

The design for the nozzle for this subsonic test section is established mostly on experiential information. Since flow conditions are not as important in this design, a highly structured design is not necessary. This discussion describes the bay requirements, assembly elements, and the specifications of the subsonic system.

Low and high-speed flow systems have been designed for use with a flow heater. Each system is composed of a sudden expansion chamber, a test section and an exhaust duct. The low-speed system does not simulate a particular component in a scramjet, but provides, instead, flow in which to evaluate measurement techniques. The high-speed system will provide a flow that qualitatively approximates the gas conditions exiting a scramjet combustion chamber. Between the heater and each test section is a sudden expansion chamber that serves as a barrier to prevent the heated electrical currents from entering the test section. Following each test section is the exhaust duct, which transports the gas safely to the atmosphere. The exhaust duct also provides indirect control of the flow conditions in the test section. Figure 3.1 shows the flow heater and test section of the wind tunnel.
The conditions at the exit of the heater are the inlet conditions for the flow systems. The gas is assumed to be dry air as a mixture of perfect gases. The pressure and temperature at the heater exit are assumed to be 5 atmospheres (absolute static) and 2800K, respectively. For these conditions, the code provided by Heiser and Pratt (1994) estimates the following gas values:

- gas constant: 290 \( \text{m/s}^2/\text{K} \)
- ratio of specific heats: 1.22
- sound speed: 990 m/s
- mole fractions:
  - \( \text{N}_2 \): 0.77
  - \( \text{O}_2 \): 0.19
  - \( \text{NO} \): 0.03
  - \( \text{N} \) and \( \text{O} \), together: 0.01
The gas flows through the 38 mm internal diameter heater at a design rate of 0.09 kg/s. Assuming uniform flow at the heater exit, then \( \dot{m} = \rho AV = \frac{p}{RT} AV \) provides the heater exit velocity, which is 120 m/s. The corresponding Mach number is 0.12.

**Construction Features**

Graphite is the principal material for the flow systems. The graphite is 99.9% carbon with a maximum grain size of 0.76 mm. Uncoated sapphire windows in the walls of the test section provide optical access. These windows are 50.8 mm in diameter, 3.15 mm thick and have a surface accuracy of 2 waves per 25.4 mm at 632.8 nm over 90% of their aperture. They will be film-cooled with air to keep them below their 2300K melting temperature. Boron nitride and epoxy glass provide electrical and thermal insulation at critical locations.

Thermocouples will monitor wall temperatures at key locations in the expansion chamber, test section and exhaust duct. These thermocouples will be ungrounded to provide electrical isolation. They have a temperature limit of 1800K.

**Low Speed System Details**

A sketch of the expansion chamber, test section and exhaust transition of the low speed system appears below.
Expansion

The expansion chamber ("expansion" because of geometry, not because of pressure) begins as a sudden expansion from the 38 mm diameter heater duct to an 89 mm diameter axisymmetric backward facing step. A conical divergence with a final diameter of 127 mm immediately follows the backstep. The divergence angle is 4°.

The sudden expansion forms an axisymmetric backward facing step for the flow leaving the heater duct. The flow separates at this backstep, but will ultimately reattach to the duct walls if the downstream pressure is sufficiently high. Experimental studies of subsonic flow past axisymmetric backsteps indicate that reattachment occurs approximately 8 step heights downstream for moderate expansion ratios. (Drewry (1978); Yang and Yu (1983), Gould, et al. (1990)). The step height chosen for this design is 25 mm. Reattachment should thus occur about 200 mm downstream of the step if the large duct remains at the initial 89 mm diameter. The 4° conical divergence provides a very gradual enlargement to the final 127 mm diameter, so that the actual reattachment distance will not be much different.
Recovery and Test Section

A recovery section follows the expansion. This section allows the boundary layer to recover from the reattachment process. The basis for the recovery length is the distance required for the turbulent shape factor $H = \frac{\delta^*}{\theta}$ and the Clauser shape factor

$$G = \sqrt{\frac{2}{C_f}} \frac{H-1}{H}$$

(3.1)

to become effectively invariant with axial position. Adams, Johnston and Eaton (1984) observe $H$ and $G$ to level off at

$$\frac{X-x_r}{x_r} \approx 2$$

(3.2)

where $X$ is the required distance downstream from the step and $x_r$ is the reattachment length. Bradshaw and Wong (1972) observe leveling of $H$ and $G$ for a planar double-sided sudden expansion at

$$\frac{X}{h} \approx 22$$

(3.3)

where $h$ is the step height. The former criterion gives $X \approx 24h$, while the latter gives $X \approx 22h$. Figure 3.3 shows the reattachment process. Using these values as guides, the test section windows are located 625 mm, 25 step heights, downstream of the backstep.
Figure 3.3 Reattachment

The test section is a short, constant diameter section fitted with the sapphire windows. The windows are recessed in the duct wall so that the flow passes over shallow cavities at the window locations. Relatively cold outside air will be blown into these cavities to cool the windows.

Transition and Exhaust

Following the test section is a transition section that brings the flow into the exhaust duct. The transition profile is a cubic curve based on guidelines from Szczeniowski (1943).

The exhaust duct needs to be approximately 4 m long in order to transport the hot gas safely to the outside. In addition to carrying the hot gas away from personnel and equipment, the exhaust duct can also provide flow control through frictional effects. To illustrate, consider a duct that is 4 m long, 28 mm in diameter with a nominal friction factor of 0.02. Furthermore, assume that the gas enters the exhaust at the rate of 0.09 kg/s with \( M = 0.24 \) and \( \gamma = 1.22 \). These flow conditions are consistent with the assumed heater exit conditions described above. The pressure required to drive this flow so that it exits into the atmosphere at a pressure nearly equal to atmospheric can be estimated using
Fanno flow analysis. From Hodge and Koenig (1995) this analysis indicates that 5 atm (abs static) pressure at the duct entrance will produce sonic flow at the duct exit with 1.1 atm (abs static) pressure. Thus, friction forces the duct inlet pressure to be high, which, in turn, requires the test section Mach number to be low, as desired.

The present design uses 4 m of 25 mm diameter graphite tubing for the exhaust duct. This diameter is 10% less than the above example to allow for gradual ablation and erosion by the hot gas.
CHAPTER IV

CONCLUDING REMARKS

The chief objectives for this work were accomplished. Both supersonic and subsonic nozzles were designed for the wind tunnel currently being constructed. For the supersonic design, the method of characteristics program tracked the streamline for the shape of the upper wall of the nozzle. However, there are multiple areas that this program and analysis did not cover. Even though the streamline tracked is like the edge of the boundary layer, boundary layers were not accounted for. This analysis is only for planar flow, and it only covers diverging sections. Future work should be extended to include axisymmetric flow and converging sections. Finally, the initial value line used is vertical. Efforts could be extended to a better approximation using Sauer’s method.

As far as the subsonic system is concerned, there were no elaborate design methods used for the subsonic section due to the short amount of time. The key goals were to avoid flow separation regions and verify that the boundary layer would settle down and recover to a normal growth pattern. These things were achieved. The wind tunnel is nearing completion, and will soon be tested. More complex design techniques could be used for another subsonic section, and the designed supersonic nozzle could be built and added to the current system.
REFERENCES


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APPENDIX B

MATHCAD NOZZLE DESIGN PROGRAM
\[ \gamma := 1.4 \quad R := 1716 \quad M_c := 3 \quad n_c := 15 \quad x_0 := 0 \quad y_0 := 1 \]

\[ \text{PM}(x) := \sqrt{\frac{\gamma + 1}{\gamma - 1}} \cdot \operatorname{atan} \left( \frac{\gamma - 1}{\gamma + 1} \left( x^2 - 1 \right) \right) - \operatorname{atan} \left( \sqrt{x^2 - 1} \right) \]

\[ \text{nd}(n_c) := \begin{align*}
\text{nd}_{1,1} & \leftarrow 1 \\
\text{for } i_w & \in 1..n_c \\
\text{nd}_{2,i_w} & \leftarrow \text{nd}_{1,i_w} + [n_c - (i_w - 1)] \\
\text{nd}_{1,i_w + 1} & \leftarrow \text{nd}_{2,i_w} + 1
\end{align*} \]

\[ \text{cw} \leftarrow \text{submatrix}(\text{nd}, 1, 2, 1, n_c) \]

\[ \text{wave}(n_c, c_w) := \begin{align*}
\text{for } i_p & \in 1..c_w_{2,n_c} \\
i_w & \leftarrow 1 \\
\text{while } i_p > c_w_{2,i_w} \\
i_w & \leftarrow i_w + 1 \\
wpt_{i_p} & \leftarrow i_w
\end{align*} \]

\[ wpt \leftarrow \text{wave}(n_c, c_w) \]

\[ a(i_p, i_w) := i_p - [n_c - (i_w - 2)] \]

\[ \text{feed}_a(n_c, c_w, wpt) := \begin{align*}
\text{for } i_w & \in 2..n_c \\
b & \leftarrow c_w_{1,i_w} \\
e & \leftarrow c_w_{2,i_w} - 1 \\
\text{for } i_p & \in b..e \\
\text{fa}_{i_p} & \leftarrow a(i_p, wpt_{i_p})
\end{align*} \]

\[ \text{fa} \leftarrow \text{feed}_a(n_c, c_w, wpt) \]

\[ \text{feed}_b(n_c, c_w) := \begin{align*}
\text{for } i_w & \in 1..n_c \\
b & \leftarrow c_w_{1,i_w} + 1 \\
e & \leftarrow c_w_{2,i_w} \\
\text{for } i_p & \in b..e \\
\text{fb}_{i_p} & \leftarrow i_p - 1
\end{align*} \]

\[ \text{fb} \leftarrow \text{feed}_b(n_c, c_w) \]
npt := cw\_2,\_nc \quad \text{wall}_i := \text{npt} + 1 \quad \text{iw} := 2..\text{nc} \quad \text{wall}_{iw} := cw\_2,\_iw-1 \quad \text{iw} := 1..\text{nc}

\text{fa}(cw\_2,\_iw) := \text{wall}_{iw} \quad \text{fa}_{iw} := \text{npt} + 1

\nu_{e\_d} := \frac{180}{\pi} \cdot \text{PM}(M_e) \quad \theta_{0\_d} := \frac{1}{2} \nu_{e\_d} \quad \theta_{1d1} := \theta_{0\_d} - \text{floor}(\theta_{0\_d}) \quad \Delta \theta := \frac{\text{floor}(\theta_{0\_d})}{\text{nc} - 1}

i := 2..\text{nc} \quad \theta_{0d1} := \theta_{0d1} + \Delta \theta \quad \theta_{0d+1} := \theta_{0d+1} \quad \nu_{d1} := \theta_{d1} + \nu_{d1} \quad \text{Kpd} := \theta_{d1} - \nu_{d1}

\text{one}_1 := \theta_{d1} \quad \text{one}_2 := \nu_{d1}

\text{Km\_ns(nc, cw, Kmd)} := \text{for} \ i w \in 2..\text{nc} \quad \text{Kmd} := \text{Km\_ns(nc, cw, Kmd)}

\text{b} \leftarrow \text{cw}_1,\_iw
\text{e} \leftarrow \text{cw}_2,\_iw - 1
\text{for} \ \text{ip} \in \text{b}..\text{e}
\text{f} \leftarrow \text{fa}_{\text{ip}}
\text{Kmd}_{\text{ip}} \leftarrow \text{Kmd}_{\text{f}}
\text{Kmd}_{(cw\_2,\_iw)} \leftarrow \nu_{e\_d}
\text{Kmd}

\text{Kpd(nc, cw, fa, Kmd)} := \text{for} \ i w \in 2..\text{nc} \quad \text{Kpd} := \text{Kpd(nc, cw, fa, Kmd)}

\text{c} \leftarrow \text{cw}_1,\_iw
\text{f} \leftarrow \text{fa}_{\text{c}}
\text{Kpd}_{\text{c}} \leftarrow -\text{Kmd}_{\text{f}}
\text{b} \leftarrow \text{cw}_1,\_iw + 1
\text{e} \leftarrow \text{cw}_2,\_iw
\text{for} \ \text{ip} \in \text{b}..\text{e}
\text{Kpd}_{\text{ip}} \leftarrow \text{Kpd}_{\text{ip}-1}
\text{Kpd}

\text{centerline(nc, cw, Kmd, one)} := \theta_{v1,\_1} \leftarrow \text{one}_1
\text{cl} := \text{centerline(nc, cw, Kmd, one)}
\theta_{v2,\_1} \leftarrow \text{one}_2
\text{for} \ i w \in 2..\text{nc}
\text{c} \leftarrow \text{cw}_1,\_iw
\theta_{v1,\_iw} \leftarrow 0
\theta_{v2,\_iw} \leftarrow \text{Kmd}_{\text{c}}
\theta_{v}
interior(nc, cw, Kmd, Kpd, fa, fb) :=
for iw ∈ 2.. nc − 1
b ← cw1, iw + 1
e ← cw2, iw − 1
for ip ∈ b.. e
θ1, ip ← \left(\frac{1}{2}\right)\left[\text{Kmd}(fa_{ip}) + \text{Kpd}(fb_{ip})\right]
θ2, ip ← \left(\frac{1}{2}\right)\left[\text{Kmd}(fa_{ip}) - \text{Kpd}(fb_{ip})\right]
θv

int := interior(nc, cw, Kmd, Kpd, fa, fb)
walk(nc, cw, int, cl) :=
θ1, 1 ← 01d_{nc}
θ2, 1 ← v1d_{nc}
θv1, nc ← cl1, nc
θv2, nc ← cl2, nc
for iw ∈ 2.. nc − 1
s ← cw2, iw − 1
θ1, iw ← int1, s
θ2, iw ← int2, s
θv

npt := cw2, nc
npt = 135 i := 1.. npt − 3 θd_i := int1, i v_d_i := int2, i j := 1.. nc
θd_j := 01d_j v_d_j := v1d_j θd(cw1, j) := cl1, j v_d(cw1, j) := cl2, j θd(cw2, j) := wll1, j v_d(cw2, j) := wll2, j

Given PM(x) - nu = 0 Mnu(x, nu) := Find(x)

j := 1.. npt x := \frac{1}{2}(M_c - 1) + 1 M_j := Mnu\left(x, \frac{\pi}{180} \cdot v_d_j\right) μ_d_j := \text{asin}\left(\frac{1}{M_j}\right) \cdot \frac{180}{\pi}
v_j := \frac{\pi}{180} v_d_j θ_j := \frac{\pi}{180} θd_j μ_j := \frac{\pi}{180} μ_d_j θμ_{1, j} := θ_j θμ_{2, j} := μ_j
slope_ac(nc,cw,θμ,fa) := for i ∈ 1..cw2,nc

θ_i ← θμ1,i
µ_i ← θμ2,i

for i ∈ 1..nc
sac_i ← tan(θ_i - µ)

for i ∈ 2..nc
b ← cw1,i
e ← cw2,i - 1

for j ∈ b..e
sac_j ← tan[\frac{1}{2} \left[ \theta(fa_j) - \mu(fa_j) \right] + (θ_j - µ_j) ]

sac(cw2,1) ← tan[θ(cw2,1)]

for i ∈ 2..nc
j ← cw2,i
sac_j ← tan[\frac{1}{2}(θ_j + θ_{j-1})]

sac := slope_ac(nc,cw,θμ,fa)

slope_bc(nc,cw,θμ) := for i ∈ 1..nc

b ← cw1,i + 1
e ← cw2,i

for j ∈ 1..cw2,nc
θ_j ← θμ1,j
µ_j ← θμ2,j

for j ∈ b..e
sbe_j ← tan[\frac{1}{2} \left[ \theta_{j-1} + µ_{j-1} \right] + (θ_j + µ_j) ]

sbc := slope_bc(nc,cw,θμ)
\[ \text{xy}_1, i \leftarrow \frac{-1}{\text{sac}_i} \]

\[ \text{xy}_2, i \leftarrow 0 \]

for \( i \in 1..\text{nc} + 1 \)

\[ \text{xy}_a, i \leftarrow x0 \]

\[ \text{xy}_a, i \leftarrow y0 \]

for \( i \in 2..\text{nc} + 1 \)

\[ \text{x}_a, i \leftarrow \text{xy}_a, i \]

\[ \text{y}_a, i \leftarrow \text{xy}_a, i \]

\[ \text{x}_b, i \leftarrow \text{xy}_1, i-1 \]

\[ \text{y}_b, i \leftarrow \text{xy}_2, i-1 \]

\[ \text{x}_c, i \leftarrow \frac{\text{ya}_i - \text{yb}_i + \text{sbc}_i \cdot \text{xb}_i - \text{sac}_i \cdot \text{xa}_i}{\text{sbc}_i - \text{sac}_i} \]

\[ \text{yc}_i \leftarrow \text{ya}_i + \text{sac}_i \cdot (\text{x}_c_i - \text{xa}_i) \]

\[ \text{xy}_1, i \leftarrow \text{xc}_i \]

\[ \text{xy}_2, i \leftarrow \text{yc}_i \]

\[ \text{xy} \]

\[ \text{xy}_i \leftarrow \text{crd}(\text{nc}, \text{cw}, \text{sac}, \text{sbc}) \]
coord(nc, cw, xyi, sac, sbc, fa) :=

<table>
<thead>
<tr>
<th>i ∈ 2..nc</th>
</tr>
</thead>
<tbody>
<tr>
<td>c ← cw1,i</td>
</tr>
<tr>
<td>f ← fa_c</td>
</tr>
<tr>
<td>xa ← xyi_1,f</td>
</tr>
<tr>
<td>ya ← xyi_2,f</td>
</tr>
<tr>
<td>xyi_1,c ← xa_c - \frac{ya_c}{sac_c}</td>
</tr>
<tr>
<td>xyi_2,c ← 0</td>
</tr>
<tr>
<td>b ← cw1,i + 1</td>
</tr>
<tr>
<td>e ← cw2,i</td>
</tr>
</tbody>
</table>

for j ∈ b..e

<table>
<thead>
<tr>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
<td>f ← fa_j</td>
</tr>
<tr>
<td>xa ← xyi_1,f</td>
</tr>
<tr>
<td>ya ← xyi_2,f</td>
</tr>
<tr>
<td>xb ← xyi_1,j-1</td>
</tr>
<tr>
<td>yb ← xyi_2,j-1</td>
</tr>
<tr>
<td>xc_j ← \frac{ya_j - yb_j + sbc_j \cdot xb_j - sac_j \cdot xa_j}{sbc_j - sac_j}</td>
</tr>
<tr>
<td>yc_j ← ya_j + sac_j \cdot (xc_j - xa_j)</td>
</tr>
<tr>
<td>xyi_1,j ← xc_j</td>
</tr>
<tr>
<td>xyi_2,j ← yc_j</td>
</tr>
</tbody>
</table>

xyi := coord(nc, cw, xyi, sac, sbc, fa)
plot(nc, cw, fa) :=
  k ← 0
  for i ∈ 1..nc
    k ← k + 1
    plt_k ← cw_1, i
    for j ∈ 1..i
      k ← k + 1
      plt_k ← fa(plt_k_{j-1})
  for j ∈ 1..i
    k ← k + 1
    plt_k ← cw_2, j
  for j ∈ 1..nc + 1 - i
    k ← k + 1
    plt_k ← plt_k_{j-1} - 1
  plt := plot(nc, cw, fa)

cmr := npt + 1
x_{1, cmr} := x_0
x_{2, cmr} := y_0
j := 1..rows(plt)
xplt_j := x_{1, j}
yplt_j := x_{2, j}
nos := 6
xp := x_{1, nos} + 10^{-6}
yp := x_{2, nos}
att(\theta, xy) :=
  xys_{1, nos} ← xp
  xys_{2, nos} ← yp
  for i ∈ nos..2
    ss ← \tan[0.5(\theta_{1, i} + \theta_{1, i-1})]
    xys_{1, i-1} ← \text{frac}{xys_{2, i} - ss \cdot xys_{1, i} - 1}{sac_{i-1} - ss}
    xys_{2, i-1} ← sac_{i-1} \cdot xys_{1, i-1} + 1
  xys := att(\theta, xy)

nt := 5
ntc := nt + nos
ntc = 11
\text{att2}(\theta, \xy) := R \leftarrow \sqrt{\tan(\theta_1, 1) \frac{\xy_1}{\tan(\theta_1, 1)}} + 1 \cdot \frac{\xy_1}{\tan(\theta_1, 1)} \quad \xy := \text{att2}(\theta, \xy)
\text{xyw} := \text{augment}(\xy, \xy)
\text{nb} := 0.5 \cdot \text{nc}^2 + 0.5 \cdot \text{nc} - 1

\text{tri} := \text{tri}(\text{nc})
\text{tribox} := \text{for } i \in 1..\text{nc} - 2 \quad \text{tri} := \text{tribox}(\text{nc})
\text{for } i \in 1..\text{nc} - 2
\begin{align*}
k &\leftarrow 1 \\
k_{i+1} &\leftarrow k_i + \lfloor \text{nc} - (i - 1) \rfloor
\end{align*}
\text{wbox} := \text{wbox}(\text{nc})
\text{for } i \in 1..\text{nc} - 2 \quad \text{wbox} := \text{wbox}(\text{nc})
\begin{align*}
b &\leftarrow \text{nc} \\
b_{i+1} &\leftarrow b_i + (\text{nc} - i)
\end{align*}

\text{box} := \text{for } i \in 1..\text{nb}
\begin{align*}
\text{box}_{i, 1} &\leftarrow i + \text{nc} \\
\text{box}_{i, 5} &\leftarrow \text{box}_{i, 1} \\
\text{box}_{i, 4} &\leftarrow \text{nc} + i + 1 \\
\text{box}_{i, 3} &\leftarrow f_{\text{a}}(\text{box}_{i, 4}) \\
\text{box}_{i, 2} &\leftarrow f_{\text{b}}(\text{box}_{i, 3})
\end{align*}
\text{for } j \in 1..\text{rows}(	ext{tri})
\begin{align*}
\text{box}_{(\text{tri}), 1} &\leftarrow 0 \\
\text{box}_{(\text{tri}), 5} &\leftarrow 0
\end{align*}
\[
\text{tann}(\text{box, xy}) := \begin{aligned}
&\text{for } j \in 1..\text{nb} \\
&\text{for } i \in 1..4 \\
&\quad \text{box}_{j,i} \leftarrow 2 \text{ if } \text{box}_{j,i} = 0 \\
&\quad \text{box}_{j,5} \leftarrow 1 \text{ if } \text{box}_{j,5} = 0 \\
&\quad \Delta x \leftarrow x_{y1,\text{box}_{j,i-1}} - x_{y1,\text{box}_{j,i}} \\
&\quad \Delta y \leftarrow x_{y2,\text{box}_{j,i-1}} - x_{y2,\text{box}_{j,i}} \\
&\quad \alpha_i \leftarrow \text{atan2}(\Delta x, \Delta y) \\
&\quad \alpha_i \leftarrow 2\pi + \alpha_i \text{ if } \alpha_i < 0 \\
&\quad n_i \leftarrow \alpha_i + \frac{\pi}{2} \\
&\quad \tan_{n,j,i} \leftarrow \tan(n_i)
\end{aligned}
\]

\[
\begin{aligned}
\text{tan}_n := \text{tann}(\text{box, xy})
\end{aligned}
\]

\[
\text{tan}\alpha(\text{box, xy}) := \begin{aligned}
&\text{for } j \in 1..\text{nb} \\
&\text{for } i \in 1..4 \\
&\quad \text{box}_{j,i} \leftarrow 2 \text{ if } \text{box}_{j,i} = 0 \\
&\quad \text{box}_{j,5} \leftarrow 1 \text{ if } \text{box}_{j,5} = 0 \\
&\quad \Delta x \leftarrow x_{y1,\text{box}_{j,i-1}} - x_{y1,\text{box}_{j,i}} \\
&\quad \Delta y \leftarrow x_{y2,\text{box}_{j,i-1}} - x_{y2,\text{box}_{j,i}} \\
&\quad \alpha_i \leftarrow \text{atan2}(\Delta x, \Delta y) \\
&\quad \alpha_i \leftarrow 2\pi + \alpha_i \text{ if } \alpha_i < 0 \\
&\quad \tan_{\alpha,j,i} \leftarrow \tan(\alpha_i)
\end{aligned}
\]

\[
\begin{aligned}
\text{tan}_\alpha := \text{tan}\alpha(\text{box, xy})
\end{aligned}
\]

\[
\text{xcrn} := \begin{aligned}
&\text{for } i \in 1..\text{nb} \\
&\text{for } j \in 1..5 \\
&\quad \text{box}_{i,j} \leftarrow \text{box}_{i,j} \\
&\text{for } m \in 1..\text{rows(tri)} \\
&\quad \text{box}_{(\text{tri}_m),1} \leftarrow \text{npt} + 1 \\
&\quad \text{box}_{(\text{tri}_m),5} \leftarrow \text{npt} + 1 \\
&\text{for } k \in 1..5 \\
&\quad \text{xcrn}_{i,k} \leftarrow x_{y1,\text{box}_{i,k}}
\end{aligned}
\]

\[
\text{xcrn}
\]
ycrn := for i ∈ 1..nb
  for j ∈ 1..5
    boxij, ← boxij,
  for m ∈ 1..rows(tri)
    box{tri}_m, 1 ← npt + 1
    box{tri}_m, 5 ← npt + 1
  for k ∈ 1..5
    ycrnk_i, k ← xy2 boxik
    ycrnk_i, k ← 0 if xy2, boxik, k = 1
  ycrnr

nx(box, xy) := for j ∈ 1..nb
  for i ∈ 1..4
    boxji, i ← 2 if boxji, i = 0
    boxji, 5 ← 1 if boxji, 5 = 0
    Δx ← xy1 boxji, i+1 − xy1 boxji,
    Δy ← xy2 boxji, i+1 − xy2 boxji,
    αi ← atan2(Δx, Δy)
    αi ← 2π + αi if αi < 0
    nj ← αi + π
    xji, i ← xcrnr_ji, i + 0.1 cos(nj)

xn := nx(box, xy)
\[\text{ny}(\text{box}, \text{xy}) := \begin{align*}
\text{for } j \in 1..\text{nb} \quad & \quad \text{yn} := \text{ny}(\text{box}, \text{xy}) \\
\text{for } i \in 1..4 \quad & \quad \begin{align*}
\text{box}_{j,i} & \leftarrow 2 \text{ if } \text{box}_{j,i} = 0 \\
\text{box}_{j,5} & \leftarrow 1 \text{ if } \text{box}_{j,5} = 0 \\
\Delta x & \leftarrow \text{xy}_1, \text{box}_{j,i+1} - \text{xy}_1, \text{box}_{j,i} \\
\Delta y & \leftarrow \text{xy}_2, \text{box}_{j,i+1} - \text{xy}_2, \text{box}_{j,i} \\
\alpha_i & \leftarrow \text{atan2}(\Delta x, \Delta y) \\
\alpha_i & \leftarrow 2\pi + \alpha_i \text{ if } \alpha_i < 0 \\
n_{j,i} & \leftarrow \alpha_i + \frac{\pi}{2} \\
y_{j,i} & \leftarrow \text{ycrn}_{j,i} + 0.1 \sin(n_{j,i})
\end{align*}
\end{align*}\]

\[\text{Dot}(\text{xp}, \text{yp}) := \begin{align*}
\text{for } i \in 1..\text{nb} \quad & \quad \begin{align*}
\text{for } j \in 1..4 \quad & \quad \begin{align*}
\text{Lcn}_{i,j} & \leftarrow \sqrt{(\text{x}_{1,i,j} - \text{xcrn}_{i,j})^2 + (\text{y}_{1,i,j} - \text{ycrn}_{i,j})^2} \\
\text{x}_{i,j} & \leftarrow \frac{\text{yp} - \text{ycrn}_{i,j} + \tan_b \text{xcrn}_{i,j} - \tan_n \text{x}_{i,j} \cdot \text{xp}}{\tan_b \text{x}_{i,j} - \tan_n \text{x}_{i,j}} \\
\text{yi}_{i,j} & \leftarrow \text{yp} + \tan_n \text{x}_{i,j} \cdot (\text{x}_{i,j} - \text{xp}) \\
\text{Lpi}_{i,j} & \leftarrow \sqrt{(\text{x}_{i,j} - \text{xp})^2 + (\text{yi}_{i,j} - \text{yp})^2}
\end{align*} \\
\text{for } j \in 1..5 \quad & \quad \begin{align*}
\text{dot}_{i,j} & \leftarrow \frac{(\text{x}_{i,j} - \text{xp}) (\text{x}_{i,j} - \text{xcrn}_{i,j}) + (\text{yi}_{i,j} - \text{yp}) (\text{yn}_{i,j} - \text{ycrn}_{i,j})}{\text{Lpi}_{i,j} \cdot \text{Lcn}_{i,j}} \text{ if } j < 5 \\
\text{dot}_{i,j} & \leftarrow \sum_{j=1}^{4} \text{dot}_{i,j} \text{ otherwise}
\end{align*}
\end{align*} \\
\text{inside} & \leftarrow i \text{ if } \text{dot}_{i,j} = 4
\end{align*}\]

\[\text{dot} := \text{Dot}(\text{xp}, \text{yp})\]
\[\text{Int(Dot)} :=\]

\[
\begin{align*}
A & \leftarrow \text{box}_{\text{dot}, 2} \\
B & \leftarrow \text{box}_{\text{dot}, 3} \\
C & \leftarrow \text{box}_{\text{dot}, 4} \\
D & \leftarrow \text{box}_{\text{dot}, 5} \\
\begin{align*}
b_1 & \leftarrow \frac{x y_2, C - x y_2, D}{x y_1, C - x y_1, D} \\
d_1 & \leftarrow \frac{x y_2, B - x y_2, A}{x y_1, B - x y_1, A} \\
a_1 & \leftarrow x y_2, C - b_1 x y_1, C \\
c_1 & \leftarrow x y_2, B - d_1 x y_1, B \\
\text{xyint}_{1, 1} & \leftarrow \frac{a_1 - c_1}{d_1 - b_1} \\
\text{xyint}_{2, 1} & \leftarrow a_1 + b_1 \cdot \text{xyint}_{1, 1} \\
b_2 & \leftarrow \frac{x y_2, D - x y_2, A}{x y_1, D - x y_1, A} \\
d_2 & \leftarrow \frac{x y_2, C - x y_2, B}{x y_1, C - x y_1, B} \\
a_2 & \leftarrow x y_2, D - b_2 x y_1, D \\
c_2 & \leftarrow x y_2, C - d_2 x y_1, C \\
\text{xyint}_{1, 2} & \leftarrow \frac{a_2 - c_2}{d_2 - b_2} \\
\text{xyint}_{2, 2} & \leftarrow a_2 + b_2 \cdot \text{xyint}_{1, 2} \\
\text{xyint} & \end{align*}
\]
\[
\text{phi}(x_p, y_p, \text{Dot}, \text{Int}) :=
\]

\[
x_{\text{new}1} \leftarrow x_p
\]

\[
x_{\text{new}2} \leftarrow y_p
\]

for \( j \in 1..333 \)

\[
D_{T_{j,1}} \leftarrow \text{Dot}(x_{\text{new}1}, y_{\text{new}2})
\]

\[
\text{INT} \leftarrow \text{Int}(D_{T_{j,1}})
\]

\[
A \leftarrow \text{box}(D_{T_{j,1}}, 2)
\]

\[
B \leftarrow \text{box}(D_{T_{j,1}}, 3)
\]

\[
C \leftarrow \text{box}(D_{T_{j,1}}, 4)
\]

\[
D \leftarrow \text{box}(D_{T_{j,1}}, 5)
\]

\[
\Delta x_{y1,1} \leftarrow x_{y1,C} - \text{INT}_{1,1}
\]

\[
\Delta x_{y2,1} \leftarrow x_{y2,C} - \text{INT}_{2,1}
\]

\[
\Delta x_{y1,2} \leftarrow x_{\text{new}1} - \text{INT}_{1,1}
\]

\[
\Delta x_{y2,2} \leftarrow x_{\text{new}2} - \text{INT}_{2,1}
\]

\[
\Delta x_{y1,3} \leftarrow x_{y1,B} - \text{INT}_{1,1}
\]

\[
\Delta x_{y2,3} \leftarrow x_{y2,B} - \text{INT}_{2,1}
\]

\[
\Delta x_{y1,4} \leftarrow x_{y1,D} - \text{INT}_{1,2}
\]

\[
\Delta x_{y2,4} \leftarrow x_{y2,D} - \text{INT}_{2,2}
\]

\[
\Delta x_{y1,5} \leftarrow x_{\text{new}1} - \text{INT}_{1,2}
\]

\[
\Delta x_{y2,5} \leftarrow x_{\text{new}2} - \text{INT}_{2,2}
\]

\[
\Delta x_{y1,6} \leftarrow x_{y1,C} - \text{INT}_{1,2}
\]

\[
\Delta x_{y2,6} \leftarrow x_{y2,C} - \text{INT}_{2,2}
\]

for \( i \in 1..6 \)

\[
\phi_i \leftarrow \text{atan2}(\Delta x_{y1,i}, \Delta x_{y2,i})
\]

\[
\phi_i \leftarrow 2\pi + \phi_i \text{ if } \phi_i < 0
\]

wall \( \leftarrow 0 \)

for \( k \in 1..nc - 1 \)

wall \( \leftarrow 1 \text{ if } D_{T_{j,1}} = \text{wbox}_k \)

\[
K_p \leftarrow K_p + \frac{K_{pd_B} - K_{pd_C}}{\phi_3 - \phi_1} (\phi_2 - \phi_1)
\]

\[
K_m \leftarrow K_m + \frac{K_{md_C} - K_{md_D}}{\phi_6 - \phi_4} (\phi_5 - \phi_4)
\]

\[
00 \leftarrow \frac{1}{(K_p + K_m) - \frac{\pi}{2}}
\]
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.717</td>
<td>0.511</td>
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<td>1.416</td>
<td>324.865</td>
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\[ b_{xy} = \]

\[ ii := 1. \ldots \text{rows}(b_{xy}) \quad jj := 1. \ldots \text{ntc} \quad kk := 2. \ldots 30 \]

\[ xy_{d1,1} := b_{xy_{rows}(b_{xy}),2} \quad xy_{d2,1} := b_{xy_{rows}(b_{xy}),3} \quad xy_{d1,31} := 16.5 \quad xy_{d2,31} := xy_{d2,1} \]

\[ xy_{d1, kk} := \frac{xy_{d1,31} - xy_{d1,1}}{30} + xy_{d1, kk-1} \quad xy_{d2, kk} := xy_{d2,1} \]
Note: Not Sized to Scale
\[
\begin{align*}
\text{s}(\text{xyw, bxy}) := & \quad s_1, 1 \leftarrow 0 \\
& \quad s_2, 1 \leftarrow 0 \\
& \text{for } i \in 2..\text{ntc} \\
& \quad s_1, i \leftarrow s_1, i-1 + \sqrt{(\text{xyw}_{1, i} - \text{xyw}_{1, i-1})^2 + (\text{xyw}_{2, i} - \text{xyw}_{2, i-1})^2} \\
& \quad s_2, i \leftarrow s_1, i - \text{xyw}_{1, i} \\
& \quad s_1, \text{ntc+1} \leftarrow s_1, \text{ntc} + \sqrt{(\text{bxy}_{1, 2} - \text{xyw}_{1, \text{ntc}})^2 + (\text{bxy}_{1, 3} - \text{xyw}_{2, \text{ntc}})^2} \\
& \quad s_2, \text{ntc+1} \leftarrow s_1, \text{ntc+1} - \text{bxy}_{1, 2} \\
& \text{for } j \in 2..\text{rows(bxy)} \\
& \quad s_1, i+j \leftarrow s_1, i, j-1 + \sqrt{(\text{bxy}_{j, 2} - \text{bxy}_{j-1, 2})^2 + (\text{bxy}_{j, 3} - \text{bxy}_{j-1, 3})^2} \\
& \quad s_2, i+j \leftarrow s_1, i, j - \text{bxy}_{j, 2} \\
\end{align*}
\]

\[
S := \text{s(xyw, bxy)}
\]

\[
m := \text{ntc} + 1..\text{rows(bxy)} + \text{ntc}
\]

\[
\text{Given} \quad \text{PM}(x) - \text{nu} = 0 \quad \text{Mnu}(x, \text{nu}) := \text{Find(x)}
\]

\[
k := 1..\text{rows(bxy)} \quad x := \frac{1}{2}(M_e - 1) + 1 \quad \text{MK}_k := \text{Mnu}(x, \text{bxy}_{k, 8})
\]
\[
\begin{align*}
ii &:= nt + 1..\text{rows(bxy)} + ntc \\
cc &:= 1..nt \quad \text{sm} := nt + 1..ntc \quad \text{ns} := ntc..ntc + \text{rows(bxy)} \\
\text{Given} & \quad PM(x) - nu = 0 \quad Mnu(x,nu) := \text{Find}(x) \\
i &:= 1..\text{ncs} \quad \theta_i := \theta_{\mu,1} \quad K_{m1} := K_{md} \cdot \frac{\pi}{180} \quad v_i := K_{m1} - \theta_i \quad x := \frac{1}{2} \left( M_{e} - 1 \right) + 1 \quad MN_i := Mnu(x,v_i) \\
M_{cc} &:= 1
\end{align*}
\]

\[
\begin{align*}
\text{Ms}(\theta_{\mu},K_{md},MN) :=
& \quad \text{for } i \in 1..nt \\
& \quad M_i \leftarrow 1 \\
& \quad \text{for } j \in 1..\text{ncs} \\
& \quad M_{i+j} \leftarrow MN_j \\
& \quad \text{for } k \in 1..\text{rows(bxy)} \\
& \quad M_{i+j+k} \leftarrow MK_k \\
M &:= \text{Ms}(\theta_{\mu},K_{md},MN)
\end{align*}
\]