A METHOD FOR THE INVESTIGATION OF THE AEROELASTIC BEHAVIOR
OF VERY HIGH ASPECT RATIO CYLINDERS

By

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A METHOD FOR THE INVESTIGATION OF THE AEROELASTIC BEHAVIOR
OF VERY HIGH ASPECT RATIO CYLINDERS

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This document details the investigation of the aeroelastic behavior of very high aspect ratio cylinders. The difficulty in this investigation lies in the fact that there are two important length scales: the aspect ratio and the Reynolds number. The primary goal is to develop a method that properly represents both length scales during testing. The secondary goal is to develop a method that quantifies the cylinder motion. This paper describes a wind tunnel technique designed to account for the aspect ratio and the Reynolds number of a very high aspect ratio cylinder. This paper also describes the data acquisition and analysis techniques developed here to quantify the motion. As demonstrations, these techniques are then used here to study the motions of two cylinders that are the same in all ways except cross-sectional shape.
DEDICATION

I would like to dedicate this work to my family. My father, mother, brother, and sister have been and continue to be a big influence in my life.
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I would like to express my thanks to those individuals that have helped with this project and my graduate education. Dr. Keith Koenig has been a mentor and a friend during my college career, and I thank him for being my thesis director.

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\( \nu \)  \hspace{1cm} \text{kinematic viscosity} \\
\( V \)  \hspace{1cm} \text{test section flow velocity} \\
\( L \)  \hspace{1cm} \text{length of the cable} \\
\( d \)  \hspace{1cm} \text{diameter of the cable} \\
\( \text{AR} \)  \hspace{1cm} \text{aspect ratio} \\
\( \text{Re} \)  \hspace{1cm} \text{Reynolds number} \\
\( \text{St} \)  \hspace{1cm} \text{Strouhal number} \\
\( T \)  \hspace{1cm} \text{tension in the system} \\
\( W \)  \hspace{1cm} \text{weight of the cable} \\
\( \theta_1 \)  \hspace{1cm} \text{angle position of the cable due to aerodynamic drag} \\
\( \theta_2 \)  \hspace{1cm} \text{angle position of the cable due to a higher aerodynamic drag than experienced in } \theta_1 \\
\( h_o \)  \hspace{1cm} \text{the vertical distance from the tunnel floor to the laser on the cable at } V=0 \\
\( r \)  \hspace{1cm} \text{vertical distance due to sag in the cable} \\
\( \delta \)  \hspace{1cm} \text{angle of motion of the cable due to pure rotation} \\
\( \Delta \alpha \)  \hspace{1cm} \text{relative angle of motion of the cable} \\
\( \Delta x \)  \hspace{1cm} \text{distance of the laser motion on the tunnel floor} \\
\( x_{1n} \)  \hspace{1cm} \text{distance from the } V=0 \text{ position on the tunnel floor to the position beginning the laser motion for } \theta_1 \\
\( x_{1x} \)  \hspace{1cm} \text{distance from the } V=0 \text{ position on the tunnel floor to the position ending the laser motion for } \theta_1 \\
\( x_{2n} \)  \hspace{1cm} \text{distance from the } V=0 \text{ position on the tunnel floor to the position beginning the laser motion for } \theta_2 \\
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CHAPTER I

INTRODUCTION

Wind tunnel study of the aeroelastic behavior of extremely high aspect ratio cylinders is a challenging task. Because of the fact that there are two important length scales in the problem and these are several orders of magnitude apart. Aerodynamic phenomena, such as vortex shedding, primarily depend on cylinder diameter. On the other hand, structural and inertial factors are largely functions of cylinder length, which, for a very large aspect ratio, is many times greater than the diameter. Properly accounting for the effects of these disparate scales is difficult. The difficulty is compounded because the cylinder’s degrees of freedom should be unrestricted in order to study the motion of the cylinder.

The present work involves an investigation of the aeroelastic behavior of cylinders similar to some used as aboveground electric power cables. These cables have roughly elliptical cross-sections with characteristic diameters of 0.5 inches (13 mm). In typical electric power applications, they are strung between poles and towers nominally 400 feet (122 m) apart, giving aspect ratios (length/diameter) on the order of 10,000. Typical flow velocities of concern are on the order of 30 mph (45 ft/s, 14 m/s), and, for standard air, the corresponding Reynolds number based on diameter is about 12,000.
The wind tunnel used in the current study has a four foot (1.22 m) wide test section in which the maximum speed is 100 mph (150 ft/s, 46 m/s). If the test specimen spans the tunnel and the full scale aspect ratio is retained, then the test specimen diameter must be 0.005 inches (0.13 mm) for the 400 foot span case. At maximum tunnel speed the test Reynolds number is a factor of 33 less than full scale, a difference too great for meaningful data to be obtained. A method for providing proper representation of aspect ratio and Reynolds number is needed. The development of such a method is one of two primary goals of the present work.

In addition to the scaling problems there are difficulties associated with quantifying the motion of a high aspect ratio cylinder in a wind tunnel. A force balance will not provide the necessary information and will interfere with the cylinder motion. Velocity field measurements, from hot wires, pressure rakes or laser-based systems, give only an indirect indication of cylinder motion. Accelerometers add considerable asymmetrically distributed mass to the system and, thus, significantly alter the inertial properties of the cylinder. Development of a method to quantify the cylinder motion is the second primary goal of this study.

There are many studies of electric power cables and other high aspect ratio cylinders, recent examples of which are references 1-3. Dimotakis\(^1\) explored drag characteristics using wake surveys of stationary cylinders. His results are highly accurate, but do not permit determination of the actual motion a cylinder may have. Komerath \textit{et al.}\(^2\) used flow visualization and wake surveys with cylinders which were mechanically forced to oscillate at frequencies near to and far from the natural vortex shedding frequencies.
Their work provides insight on the vortex shedding processes, particularly the three-dimensional aspects of this phenomena, but do not define the natural wind-induced cylinder motion that might occur. Popplewell and Shah\(^3\) experimentally measured forces acting on stationary high aspect ratio asymmetric cylinders and from these inferred stability derivatives. Their results allow one to estimate under what situations these objects may be unstable, but, as with the other studies, the actual natural motion was not observed. Despite the many studies that have been done on long slender cylinders, apparently few, if any, have directly explored the natural wind-induced motion. It is this problem that the present study addresses.

The purpose of the present work is to develop a method for use with the Mississippi State University (MSU) low speed wind tunnel for studying flow-induced motion of a long, slender cylinder. This paper describes a wind tunnel technique designed to account for the aspect ratio and the Reynolds number of a very high aspect ratio cylinder. This paper also describes the data acquisition and analysis techniques developed here to quantify the motion. As demonstrations, these techniques are then used here to study the motions of two cylinders that are the same in all ways except cross-sectional shape.
CHAPTER II
EXPERIMENTAL DETAILS

The experiment was broken into two phases. The first phase consisted of wind tunnel testing, and the data collected from these tests were in the form of Video Home System (VHS) video. The second phase consisted of converting the VHS video signal into digital images and developing a program that would reduce the data stored in these images. This chapter will detail the development of these two processes.

Two cylinders were tested. The cylinders shared similar properties except for cross-sectional geometry. Constructed primarily of aluminum, these cylinders weighed 0.8 lb (3.6 N) and had a characteristic diameter of 0.5 inches (13 mm). The cylinders were 4 ft (1.2 m) long and had a moment of inertia of $3.58 \times 10^{-4}$ slug-inch$^2$ ($3.37 \times 10^{-6}$ kg-m$^2$) about the long axis using the characteristic diameter.

Wind Tunnel Setup

Wind Tunnel Facility

The facility used is a subsonic wind tunnel located in Patterson Laboratory operated by the Department of Aerospace Engineering at MSU. The cross-section of the tunnel is octagonal in shape. The test section shown in Figure 2.1 measures 58 inches (4.8 ft, 1.5 m) in length, 48 inches (4 ft, 1.3 m) in width, and 36 inches (3 ft, 0.914 m) in height with a cross-sectional area of 10.5 ft$^2$ ($0.973$ m$^2$). The test section has a maximum velocity of
100 mph (150 ft/s, 46 m/s). This speed restriction is imposed by the electric motor and not by the wind tunnel. Although it is possible to attain speeds higher than the maximum, sustaining these speeds is ill-advised due to the excessive current required by the motor.

The tunnel is powered by a 75 hp (56 kW), 440V, three phase electric motor. The motor spins a 10 ft (3.05 m), four blade, variable pitch propeller at 1200 rpm. The velocity of the test section is controlled by changing the pitch of the propeller blades using a 24V direct current motor driving a 7096:1 reduction gearbox.

The test section flow velocity is monitored and controlled by using a data acquisition and control system using TESTPOINT® as the software interface. The controller uses a GUI (Graphical User Interface) to control the test section velocity. More information on the data acquisition and control program may be acquired from McAllister’s paper.

Some modifications were made to the wind tunnel for this project. To allow for the cable setup, a two inch diameter hole was cut in the back side of the test section, and the door allowing access to the tunnel was left open. When observing the flow with tunnel door open, the flow along the open door side appeared to follow the path it would have had with the door closed. That is, it formed a free jet with no apparent spreading and only a very thin shear layer. Consequently, the open door had no significant effect on the experiment.

Constraints and Boundary Conditions

When conducting wind tunnel tests, the boundary conditions and constraints must be similar to the application. The cylinders to be studied are similar to aboveground, electric power transmission lines. The typical spans for these transmission lines between power
poles is 400 ft (122 m) while the characteristic wire diameter is of 0.5 inches (13 mm).

This type of application has two important length scales to consider: Reynolds number and aspect ratio. The Reynolds number and aspect ratio are defined as

\[
Re = \frac{V \cdot d}{\nu}
\]  
(2.1)

\[
AR = \frac{L}{d}
\]  
(2.2)

where \( V \) is the flow velocity, \( d \) is the diameter of the cylinder, \( L \) is the span of the cylinder, and \( \nu \) is the kinematic viscosity of the transport fluid.

For a flow velocity of 30 mph (45 ft/s, 14 m/s), for standard air (\( \nu = 15.6 \times 10^{-6} \text{ ft}^2/\text{s} \)), the Reynolds number, using the diameter as the characteristic length, is

\[
Re = \frac{V \cdot d}{\nu} = \frac{45 \text{ ft/s} \cdot 0.5 \text{ in}}{15.6 \times 10^{-6} \text{ ft}^2/\text{s} \cdot \frac{1 \text{ ft}}{12 \text{ in}}} = 12,000
\]

If a span of 400 ft (122 m) is used, the aspect ratio, using the diameter as the characteristic length, is

\[
AR = \frac{L}{d} = \frac{400 \text{ ft}}{0.5 \text{ in} \cdot \frac{1 \text{ in}}{1 \text{ ft}}} = 9600
\]

One of the constraints of the wind tunnel is that the test section has a maximum width of four feet (1.22 m). If the full scale aspect ratio were to be retained, the cylinder would have to have a diameter of 0.005 inches (0.13 mm).

\[
d = \frac{L}{AR} = \frac{400 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}}}{9600} = 0.005 \text{ in}
\]
Solving for the test section flow velocity needed to retain the Reynolds number of 12,000, the flow velocity must be

\[ V = \frac{\text{Re} \cdot \nu}{d} = \frac{12,000 \cdot 15.6 \times 10^{-6} \text{ ft}^2 / \text{s}}{0.5 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 4500 \text{ ft/s} \Rightarrow \text{Mach 4.0} \]

The velocity must be roughly a factor of 33 greater than the maximum test section velocity, or the Reynolds number must be a factor of 33 less than full scale. Since these speeds are unobtainable (and totally unreasonable due to compressibility), a scheme must be developed to simulate the full scale aspect ratio for a four foot sample with the test section flow velocities matching the full scale conditions.

A four foot sample of these cylinders is very stiff in bending and torsion, but over very long spans, however, these cylinders become very flexible. Data collected by an independent resource provided some insight to the flexibility of a type of these cylinders. A 52 ft (15.8 m) segment of the cylinder was tested using a torque arm of one foot (0.305 m) located at mid span. When a 3 lb (13.3 N) force was applied to the torque arm, the torque arm rotated 90 deg. Therefore, for very large spans the force needed to rotate the torque arm would be significantly less. So, the cylinder is highly flexible at large spans. Also, the cylinder is pulled to a tension of 1700 lb (7560 N) so that there is only 8 ft (2.44 m) of sag at the mid span of a 400 ft (122 m) line. To conclude, there are three factors that must be incorporated in the setup for the experiment: aspect ratio, flexibility, and tension (sag).

The solution to the aspect ratio and flexibility questions was to use string. String approximates high aspect ratios by providing the bending and rotational flexibility
needed. The string used here had a diameter of 0.0312 inches (0.792 mm) with a weight of 0.01642 lb/yd (0.08 N/m). The answer to the tension was a little bit more complicated.

To solve the problem of the tension load, weight was added to the open end of the string to produce a desired amount of sag. Using trigonometry and the approximation that the cylinder is a straight line from the pole connection to the sag point at the mid span, the angle from the horizontal to the cylinder is 2.3 degrees.

\[
\theta = \arctan\left(\frac{8 \text{ ft}}{200 \text{ ft}}\right) = 2.3 \text{ deg}
\]

Applying that angle to the wind tunnel model and drawing a free body diagram, the following relation can be shown (Refer to Figure 2.2)

\[
T = \frac{W}{2 \cdot \sin(2.3 \text{ deg})}
\]

where \( T \) is the tension in the string, and \( W \) is the weight of the cylinder. Applying the weight of the four foot (1.22 m) sample, 0.8 lb (3.56 N), the tension needed on either side of the cylinder was approximately 10 lb (44.4 N).

Measuring Devices to Quantify the Cylinder Motion

Careful consideration must be taken when placing measuring devices to quantify the cylinder motion. The cylinder’s degrees of freedom should be unrestricted in order to study the motion of the cylinder. A force balance effectively removes the degrees freedom because of being restrained on the sting. Velocity field measurements, from hot wires, pressure rakes or laser-based systems, give only an indirect indication of cylinder
motion. Accelerometers add considerable asymmetrically distributed mass to the system and, thus, significantly alter the inertia properties of the cylinder.

The solution was to use a small laser mounted on the cylinder and shining on the tunnel floor. As the cylinder moved, the laser spot moved with increased amplitude due to the 21 inch (1.75 ft, 0.533 m) radius from the cylinder to the floor. The spot motion could be videotaped and analyzed to infer the cylinder motion. For these experiments the laser was attached at mid-span on the downstream side of the cylinder. To aid in the laser alignment a meter stick was attached to the tunnel floor. The mass of the laser was small compared to the cylinder, so that any asymmetrically distributed mass was negligible. The laser used was a 4.2 mW single-element glass unit with a 670 nm wavelength, a 5.0 x 1.2 mm beam size at output, and a 1.2x0.3 milli-radian divergence.

**Cylinder Setup**

Using eyehooks on each end of the cylinder, both ends of the cylinder were attached to a string measuring 106 inches (8.83 ft, 2.69 m) at each side. The string passed over pulleys, and was attached to a 10 lb (44.5 N) weight at each end. Refer to Figure 2.3. Powered by a 3V power supply, the laser, located at the mid span, was attached to downstream side of the cylinder using Velcro and one-quarter inch (6.35 mm) electrical tape. The laser placement was such that with no flow the laser beam was perpendicular to the floor of the tunnel. Figure 2.4 shows a picture of the experimental setup.
Camera Setup

A VHS camcorder supported by a tripod was used to videotape the laser motion. The lights to the test section were turned off so that the laser spot was more noticeable. The camera was orientated such that the distance from the lens to the laser spot on the tunnel floor was 100 inches (8.3 ft, 2.5 m) at an angle of 12 deg from the horizontal.

The shutter speed was important during filming. At low shutter speeds, the laser spot blurred and was difficult to find. If the shutter speed was too high, there was not enough light to make out any details in the wind tunnel. A shutter speed of 1/2000 satisfied both constraints. The unadjustable frame rate of the camcorder was 30 frames per second.

The low framing rate of the camera limited the highest observable frequency to 15 Hz (one-half the camera speed). Aerodynamic phenomena, such as vortex shedding, occur at higher frequencies. The vortex shedding frequency is typically expressed in terms of

$$St = \frac{d \cdot \omega}{V}$$  \hspace{1cm} (2.4)

where $St$ is the Strouhal number, and $\omega$ is the shedding frequency in rad/s. For high aspect ratio objects at moderate Reynolds number, $St = 0.2$.

Using a Strouhal number 0.2 with a velocity and characteristic diameter of 30 mph (45 ft/s, 14 m/s) and 0.5 inches (12 mm), respectively, the vortex shedding frequency is

$$\omega = \frac{St \cdot V}{d} = \frac{216 \text{ rad}}{s} = 34 \text{ Hz}$$

This value is more than twice the limit imposed by the camera frame rate. With the use of high-speed cameras this phenomenon might be observed. If a high frame rate is desired achieved, a more efficient data reduction procedure must be devised. At 200
frames per second, a nine-second video would have 1,800 frames to capture. The current Mathcad® code utilizing a 800 MHz Pentium processor with 256 megabytes of memory requires 1.5 hr to analyze 256 frames. If 1,800 frames were to be analyzed, the time for completion would be 10.5 hr. Part of the problem is with using Mathcad®. Mathcad® runs from within Microsoft® Windows. Microsoft® Windows runs a number of programs in the background that causes programs to execute slowly. If a FORTRAN or Microsoft® Basic code were used, the processing time would decrease.

**Personnel**

The experiment required three people. One person controlled the speed of the wind tunnel using the wind tunnel control computer, and one operated the camera. The third person monitored the experiment and stayed by the wind tunnel emergency shutdown switch.

**Analysis Method**

**Converting the VHS Tape to Digital Images**

In order to reduce the data, the VHS tape had to be stored in digital format. An ATI-TV card was used. This card is a TV tuner and video input card that piggybacks onto an ATI graphics card. The software used to control the card was Adobe® Premiere v4.2. Adobe® Premiere is professionally used software that allows the user to capture still frames and movie clips.

For each test run for each cylinder, a nine second AVI (Audio Video Interleave) was captured at a frame rate of 30 frames per second. The resolution of the captured video
was 320 pixels in the horizontal direction and 240 pixels in the vertical direction or 320x240. Using the Adobe Premiere video player, each frame was saved as a bitmap, BMP, file. A BMP contains an array of pixels that describes an image. Each pixel is given a set of numbers that describe the color properties of that pixel.

Using Mathcad®, each bitmapped image was converted into an eight bit grayscale image. These images were stored in 320x240 arrays with the value of each element in the arrays ranging from 0 to 255. Zero represents black in color, and 255 represents white in color. Over 250 of these arrays were stored for one AVI file.

**Spot Locator Program**

Once the analog video has been transferred to a digital image format, the task becomes to locate the pixel position of the laser in the picture. Using Mathcad®, a program was developed to find the pixel location of the laser. This code language was chosen because post-experiment analysis of the data is easier, and there is just one software package needed, Mathcad®. The code appears in Appendix A.

Although the laser is red in color, the laser spot has a high contrast when filmed at high shutter speeds. This makes the laser spot appear pure white in the 8-bit grayscale image. Other white spots also appeared in the film. VHS tape format is prone to noise, often referred to as “snow”, when recording. Therefore, there are random white spots created by this noise. White spots also appeared as the result of extraneous reflections. The problem then becomes how to separate the white areas and distinguish the laser from the noise.
The program identifies within the image array one or more sub-arrays that contain white spots. The boundaries, or breaks, of the sub-arrays are found using two search patterns. The first search scans by columns. Starting at column number one, each element for that column is systematically searched for a value of 255. If a value of 255 is not found, the search continues to the next column. If a value of 255 is found, the program stores the column location in the first column of a “j-break” array, and the search continues to the next column to determine whether or not a 255 exists. If the column contains a 255, the search continues to the next column. If not, the previous column location is stored in the second column of the “j-break” array, a new row for the “j-break” array is created, and the search continues to find another 255.

When all columns have been searched, the program begins searching by rows. Starting at row number one, each element for that row is systematically searched for a value of 255. If a value of 255 is not found, the search continues to the next row. If a value of 255 is found, the program stores the row location to the first column of a “i-break” array, and the search continues to the next row to determine whether or not a 255 exists. If the row contains a 255, the search continues to the next row. If not, the previous row location is stored in the second column of the “i-break” array, a new row for the “i-break” array is created, and the search continues to find another 255.

Once all break points have been defined, the array is divided into sub-arrays using the break points. Each sub-array is searched to count the number of elements containing 255. The sub-array containing the greatest number 255’s is the laser spot. Visual inspection of many images revealed that the laser is about twice size of the noise, so distinguishing the laser from the noise is not a problem. The actual pixel position of the laser spot is
defined here as the centroid for the area containing the 255’s. Because the pixels are all the same size, the centroid is simply the average of the row and column positions. The average is converted to an integer to facilitate further analysis. In other words,

\[
\text{integer}(x_{\text{laser}}) = \text{integer} \left( \frac{1}{n} \cdot \sum_{i=1}^{n} x_i \right) 
\]

(2.5)

\[
\text{integer}(y_{\text{laser}}) = \text{integer} \left( \frac{1}{n} \cdot \sum_{i=1}^{n} y_i \right) 
\]

(2.6)

where \(x\) and \(y\) are the row and column positions of the 255’s, respectively.

**Example of Spot Locator Program**

To demonstrate the program, an example of a 5x5 array will be analyzed. Refer to Figure 2.5a. The search will start by defining the “j-breaks”. Starting with column 1, every element starting with row 1 is scanned until row 5. Since there are no 255 elements, the search continues to column 2. Row 2 contains a 255. Therefore, the column position is recorded, and the search continues to find a column that does not contain a 255. Refer to Figure 2.5b. Since columns 3-4 contain a 255, those columns are skipped. Since column 5 does not contain a 255, column 4 is recorded as being the end breakpoint. Refer to Figure 2.5c

The search will continue by defining the “i-breaks”. Starting with row 1, every element starting with column 1 is scanned until column 5. Since there are no 255 elements, the search continues to row 2. Column 2 contains a 255. Therefore, the row position is recorded, and the search continues to find a row that does not contain a 255. Refer to Figure 2.5d. Since rows 3-4 contain a 255, those rows are skipped. Since row 5
does not contain a 255, row 4 is recorded as being the end break point. Refer to Figure 2.5d.

Now a 3x3 sub-array is formed from the 5x5 array. Refer to Figure 2.5f. The centroid of the spot is located at row 3, column 3 of the original array.

Methods for Comparing the Motion of the Cylinders

There are a variety of ways by which the motion of different cylinders might be compared. Figure 2.6 illustrates three methods considered here. These are comparison of the actual angle of motion (δ), the relative angle of motion (Δα), or the length of the motion of the laser on the tunnel floor (Δx). Ideally, the true angle would be the best the measure of motion, but due to uncertainty in the instantaneous laser position, this angle cannot be used. The relative angle of motion is based on the angle measured by projecting the laser spot from the tunnel floor to a fixed reference point. Here the no flow position of the cylinder was chosen as the fixed reference point. Using the vertical height and the x position of the maximum and minimum range of motion, the relative angle can be found using trigonometry. The x position or the physical distance of translation, was determined by finding the pixel location in the data from the digital images and performing a transformation of coordinates from pixel to physical dimensions. The question then becomes which of the latter two methods more accurately represents the motion.

An exercise was done to compare which method would be better. Referring to Figure 2.7, the assumption is made that when subjected to flow, the cylinder only oscillates due to twisting. There is no pendulum motion. The laser is shown in three positions. The
first position is the initial no flow position where the laser is perpendicular to the tunnel flow. In the second position, the cylinder has moved downstream 16.3 deg due to aerodynamic drag. In the third position the cylinder has moved downstream 32.0 deg due to aerodynamic drag. At each of these positions the cylinder rotates about its long axis an amount \( \delta \), here equal to 5.72 deg; the cylinder does not swing, however, like a pendulum.

The angles \( \phi_{1n} \), \( \phi_{1x} \), \( \phi_{2n} \), and \( \phi_{2x} \) are given by

\[
\phi_{1n} = \theta_1 - \delta_{1/2} = 13.4 \text{ deg}
\]

\[
\phi_{1x} = \theta_1 + \delta_{1/2} = 19.1 \text{ deg}
\]

\[
\phi_{2n} = \theta_2 - \delta_{1/2} = 29.2 \text{ deg}
\]

\[
\phi_{2x} = \theta_2 + \delta_{1/2} = 34.9 \text{ deg}
\]

Using these angles, the maximum and minimum positions of the motion can be found.

\[
x_{1n} = r \cdot \sin(\theta_1) + ((h_o + r) - r \cdot \cos(\theta_1)) \cdot \tan(\phi_{1n}) = 5.9 \text{ in}
\]

\[
x_{1x} = r \cdot \sin(\theta_1) + ((h_o + r) - r \cdot \cos(\theta_1)) \cdot \tan(\phi_{1x}) = 8.2 \text{ in}
\]

\[
x_{2n} = r \cdot \sin(\theta_2) + ((h_o + r) - r \cdot \cos(\theta_2)) \cdot \tan(\phi_{2n}) = 13.6 \text{ in}
\]

\[
x_{2x} = r \cdot \sin(\theta_2) + ((h_o + r) - r \cdot \cos(\theta_2)) \cdot \tan(\phi_{2x}) = 16.5 \text{ in}
\]

The lengths, \( \Delta x_1 \) and \( \Delta x_2 \), are 2.3 inches and 3.0 inches, respectively, for which the difference is 26%. The important idea is that the same oscillation yields significantly different translation lengths when the center of oscillation changes. Comparing the same cases using the relative angle of motion,

\[
\alpha_{1n} = \arctan \left( \frac{x_{1n}}{h_o} \right) = 15.6 \text{ deg}
\]
The relative angles of rotation, $\Delta \alpha_1$ and $\Delta \alpha_2$, are 5.6 deg and 5.4 deg, respectively, for which there is only a 4.4% difference. The relative angle method gives much greater accuracy. Consequently, the method of comparison should be by the relative angle and not by the dimensional length.

**Transformation to 2-D Coordinate Space**

A method was needed to map the pixel location of the laser to a 2-D coordinate system in the plane of the floor of the wind tunnel. This was needed in order to extract the position of the laser needed for the relative angle explained in the previous section.

For this transformation it was assumed that the relation between pixel position and physical position was linear.

$$\begin{align*}
x_{\text{physical}} &= a \cdot x_{\text{pixel}} + b \cdot y_{\text{pixel}} + c \\
y_{\text{physical}} &= d \cdot x_{\text{pixel}} + e \cdot y_{\text{pixel}} + f
\end{align*}$$

(2.7)

The coefficients $a$, $b$, $c$, $d$, $e$, and $f$ were found by mapping three known positions in the image to the floor of the wind tunnel. Denoting “i” for inches and “p” for pixels, the solution to the coefficients is
Expanding the above expression,

\[
\begin{bmatrix}
\begin{array}{ccccc}
\alpha & 0 & 0 & 0 & 0 \\
0 & \beta & 0 & 0 & 0 \\
0 & 0 & \gamma & 0 & 0 \\
0 & 0 & 0 & \delta & 0 \\
0 & 0 & 0 & 0 & \epsilon \\
\end{array}
\end{bmatrix}
\begin{bmatrix}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\eta_1 \\
\eta_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
\begin{array}{c}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\eta_1 \\
\eta_2 \\
\end{array}
\end{bmatrix}
\]  

(2.8)

Fast Fourier Transform Analysis

A Fast Fourier Transform program was used in Mathcad® to analyze the dominant frequencies in the laser position and time data. These frequencies will give insight into the relationship between dominant frequencies and the natural frequencies of the cylinder. To evaluate the results from the Fast Fourier Transform, the data from the transform are used to reconstruct a signal in the time domain. This signal is compared with the original signal. The Fast Fourier Transforms outputs the signal strength of the
frequencies. The reconstructed signal is formed using these amplitudes arranged from the largest to the smallest in a sine series.

\[ x(t) = -1 \cdot \left( \sum_{a=0}^{m} \text{amp}_a \cdot \sin(f_a \cdot 2\pi \cdot t) \right) \]  

(2.10)

where \( t \) is the time, \( \text{amp} \) is the amplitude, \( f \) is the frequency in Hz for that amplitude, and \( m \) here is the number of terms used to reconstruct the signal.
CHAPTER III

DATA ANALYSIS

The data was analyzed to determine if the dominant frequencies from the Fast Fourier Transform matched the natural frequencies of the cylinder and to compare the relative motion of the two cylinders. Data were obtained at three speeds: 20 mph (30 ft/s, 9 m/s), 30 mph (45 ft/s, 14 m/s), and 40 mph (60 ft/s, 19 m/s). These speeds were chosen based on typical speeds that aboveground electrical cables experience in the northern part of the country.

**Degrees of Freedom**

There are a total of four possible types of motion that were considered as shown in Figure 3.1: vertical pendulum motion, horizontal pendulum motion, solid body rotation, and vertical motion, also known as gallop. Since the aerodynamic forces were small and the cylinders were metal, torsion and bending of the metal cylinder were negligible.

The vertical translation is a possible motion, but since the laser is attached to the cylinder such that the laser beam is perpendicular to the tunnel floor, the motion will barely be observed. If that type motion were of concern, another laser could be attached such that the laser beam shines onto the side walls of the wind tunnel. In this study, the vertical motion was not measured.
To obtain an estimate of the natural frequency for the solid body rotation, the cylinder was rotated and released. The experimental value for the solid body rotation was around 3.5 Hz.

To observe the natural frequency for the horizontal pendulum motion, one end of the cylinder was positioned further upstream than the other position and released. The test to find the value for the horizontal pendulum motion was inconclusive due to other motions induced when trying to excite that particular motion.

The equation for the natural frequency of a pendulum is

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \]  

where \( g \) is the acceleration due to gravity and \( L \) here is the sag in the string. Substituting the values into the above expression the natural vertical pendulum frequency is 1.9 Hz.

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{32.2 \text{ ft/s}^2}{2.66 \text{ in}}} = \frac{1}{2\pi} \sqrt{12 \text{ in}} = 1.91 \text{ Hz} \]

**Control Points for the 2-D Transformation**

As mentioned earlier, in order to relate pixels to physical coordinates on the tunnel floor a total of three reference points are needed. Figure 3.2 shows a sample frame that was captured from one of the six AVI files. The tunnel has several reference points that could be used. On the tunnel floor there are two openings for the force balance apparatus. The two openings consist of a rectangle and a circle that is within the rectangle. Plywood panels cover these openings. There are small gaps between these
pieces of plywood and the tunnel floor. The rectangle gap was covered with one-half inch wide black tape, but the circular gap was not taped because the crack was not big enough to have any effect on the flow. In Figure 3.3 these cracks are highlighted.

Two sets of reference points were used, one for speeds 30 ft/s and 45 ft/s, and one for 60 ft/s. For the lower speeds the reference points are the corners of the rectangle, and the intersection of the gap of the circle with the ruler. Figure 3.4a shows these control points, and Table 3.1 presents their values. For the 60 ft/s speed, a point is needed further upstream to reduce the sensitivity of the method. Figure 3.4b shows the control points used at these speeds, and Table 3.1 presents their values.

Results from the Fast Fourier Transform

Figure 3.5 shows the results from the Fast Fourier Transform for 30 ft/s, 45 ft/s, and 60 ft/s. The data processed by the Fourier analysis come directly from the pixel locations and not from the physical coordinates. The normalized amplitude from the signal response shows that there are several dominant frequencies.

Recall that cylinders A and B differ only in cross-sectional shape; their mass and moment of inertia are the same. Hence, any differences in frequency characteristics are most probably a result of differences in aerodynamic properties.

30 ft/s Comparison for Cylinder A and Cylinder B

Comparing the 30 ft/s case, the dominant frequencies occur near 1.8, 4.4, 6.0 Hz for the Cylinder A and 1.8 Hz for Cylinder B. The 1.8 Hz frequency coincides with the
vertical pendulum motion. The 4.4 Hz is close to the natural frequency for solid body rotation. The 6.0 Hz frequency could correspond to the second natural mode of rotation. It can be seen clearly that Cylinder B has partially damped some of the higher frequencies.

**45 ft/s Comparison for Cylinder A and Cylinder B**

Comparing the 45 ft/s case, the dominant frequencies occur around 6.2 Hz for Cylinder A and 3.4 Hz for Cylinder B. The rotation and pendulum motions are evident for both cylinders, but the major motion is rotation. The first mode is evident in the Cylinder B, but the second mode appears to be stronger for Cylinder A.

**60 ft/s Comparison for Cylinder A and Cylinder B**

The 60 ft/s case has the same characteristics as the 45 ft/s case. The dominant frequency occurs at 6-Hz for Cylinder A and 3.8-Hz for Cylinder B.

**Verification of Results**

The results were checked by reconstructing the signal using the frequencies and amplitudes from the Fast Fourier Transform. Figure 3.6 shows the comparison of the reconstructed signal to the measured signal. There are some frequency differences between the two signals at 30 ft/s and a 180 deg phase shift for Cylinder A at 60 ft/s. Overall, however, the reconstructed signals give confidence in the validity of the present use of the Fourier analysis.
Results from the Motion Comparison

The physical motion of the cylinders was also compared. Figures 3.7 to 3.9 show this motion for the three test speeds. A histogram also appears in each, presenting the sample distribution of the data taken from the relative of angle of motion. In addition to the histograms the standard deviation is calculated. The standard deviation of a sample population is defined by

\[ S_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2} \]  

(3.2)

where N is the number of readings, and \( X \) is a vector containing the readings. As explained by Coleman et al.\(^6\), it should be noted that \((N-1)\) occurs instead of \(N\) because the sample mean \( \bar{X} \) is used. This results in the loss of one degree of freedom since the same sample used to calculate \( S_x \) has already been used to calculate \( \bar{X} \).

30 ft/s Comparison for Cylinder A and Cylinder B

Figure 3.7 shows the cylinder motions and histograms for 30 ft/s. The relative angle motion for Cylinder A is 8.92 deg, and for Cylinder B, the motion is 7.20 deg. The sample standard deviation for Cylinder A is 1.42 deg and for Cylinder B is 1.27 deg. As can be seen, the motions are nearly the same for both objects, although B is slightly less.

45 ft/s Comparison for Cylinder A and Cylinder B

Figure 3.8 shows the cylinder motions and histograms for 45 ft/s. The relative angle motion for Cylinder A is 10.9 deg, and for Cylinder B, the motion is 6.18 deg. The sample standard deviation for Cylinder A is 1.5 deg and for Cylinder B is 1.2 deg. With
a velocity increase of 15 ft/s, there are now notable differences between the two cylinders. The motion for Cylinder A increased, whereas, the motion for Cylinder B apparently decreased. The reason for the latter behavior is not clear at this time.

**60 ft/s Comparison for Cylinder A and Cylinder B**

Figure 3.9 shows the cylinder motions and histograms for 60 ft/s. The relative angle motion for Cylinder A is 14.4 deg, and for Cylinder B, the motion is 5.67 deg. The sample standard deviation for Cylinder A is 3.1 deg and for Cylinder B is 1.1 deg. With another velocity increase of 15 ft/s, the differences between the cables are considerable. The motion for Cylinder A has increased by 32 percent and the motion for Cylinder B has surprisingly decreased again.
CHAPTER IV

CONCLUDING REMARKS

The main goals for this work were achieved. A framework for representing high aspect ratio cylinders in a wind tunnel setting and for quantifying their wind-induced motion has been created. String provides the flexibility and a small laser provides an amplified measurable signal. A tool for extracting quantitative information regarding motion from a video tape was developed. There are, certainly, questions to answer, problems to solve, and improvements needed.

One question is whether or not a string provides a realistic model. This is a question in the structural dynamics of continuous systems. The answer to the question depends upon the context of the specific application. For this investigation, the application was to approximately model aboveground, electrical cables. String appeared to provide an approximation to the problem when considering the flexibility of the particular cables and their aspect ratio. For other applications, the question must be considered.

One problem is the accurate measurement of cylinder natural frequencies. It is difficult to isolate one degree of freedom at a time. Better ways to do this are needed.

There are certainly improvements that could make the data more accurate. The transformation is very sensitive to the choice of the reference points. If there is a considerable amount of difference between the actual pixel position of the reference point
and the observed pixel position, there will be considerable error in the physical position. Therefore, more accurate reference points should be used. Increased accuracy in camera position measurement is needed. In addition, a non-linear transformation from pixel to physical position should be developed. The frame rate of the current camera system is not adequate to observe aerodynamic phenomenon such as vortex shedding. Therefore, a high frame rate recording device is needed.

Despite these issues the system developed in this project seemed to work reasonably well. Clear differences between the two test cylinders were apparent in the data, and a very interesting and unexpected behavior was observed. The slight but repeatable decrease in amplitude of motion of one cylinder as speed increased raises questions. These questions urge the refinement of the methods developed here and further experimentation.
REFERENCES


Table 3.1

Reference points used in the transformation.

<table>
<thead>
<tr>
<th></th>
<th>Pixel Coordinates</th>
<th>Physical Coordinates (inches)</th>
<th></th>
<th>Pixel Coordinates</th>
<th>Physical Coordinates (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#1</td>
<td>#2</td>
<td>#3</td>
<td>#1</td>
<td>#2</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
<td>x</td>
</tr>
<tr>
<td>Cylinder A</td>
<td>30 ft/s</td>
<td>181</td>
<td>125</td>
<td>164</td>
<td>179</td>
</tr>
<tr>
<td></td>
<td>45 ft/s</td>
<td>186</td>
<td>124</td>
<td>170</td>
<td>178</td>
</tr>
<tr>
<td></td>
<td>60 ft/s</td>
<td>111</td>
<td>121</td>
<td>236</td>
<td>193</td>
</tr>
<tr>
<td>Cylinder B</td>
<td>30 ft/s</td>
<td>133</td>
<td>118</td>
<td>113</td>
<td>171</td>
</tr>
<tr>
<td></td>
<td>45 ft/s</td>
<td>131</td>
<td>118</td>
<td>131</td>
<td>171</td>
</tr>
<tr>
<td></td>
<td>60 ft/s</td>
<td>71</td>
<td>115</td>
<td>197</td>
<td>184</td>
</tr>
</tbody>
</table>
Figure 2.1  Wind tunnel test section schematic.
Figure 2.2  Free-body diagram of the cylinder.
Figure 2.3  Schematic of the cable setup inside the test section.
Figure 2.4 Picture of the experimental setup.
Figure 2.5 Demonstration of the search program.

(a) Simple 5x5 matrix.  (b) The search routine starts by examining the columns of the matrix. Once at column 2, the first “j-break” occurs.  (c) Once the search reaches column 4 the end of the first “j-break” is defined.  (d) The search continues with the first “i-break” occurring at row 2.  (e) The search concludes with the end of the first “i-break” occurring at row 4.  (f) The program reduces the 5x5 matrix to a single matrix measuring 3x3.
Figure 2.6 Three ways to compare the motion: the angle of motion ($\delta$), the relative angle of motion ($\Delta\alpha$), and the distance of the laser travel ($\Delta x$).
Figure 2.7 Example showing what method to use when comparing the motions.
Figure 3.1 Possible types of motion of the cable.
Figure 3.2 Typical video frame from the experiment.
Figure 3.3 Gaps in tunnel floor highlighted.
Figure 3.4 Control points used for the transformation from pixels to inches.
(a) 30 ft/s and 40 ft/s, (b) 60 ft/s. Unlabeled white dot is the laser spot.
Figure 3.5 Normalized frequency response from Fast Fourier Transform. (a) 30 ft/s, (b) 45 ft/s, and (c) 60 ft/s.
Figure 3.6 Reconstructed signal for validation of the Fast Fourier Transform. Cylinder A (a) 30 ft/s, (b) 45 ft/s, (c) 60 ft/s.
Cylinder B (d) 30 ft/s, (e) 45 ft/s, (f) 60 ft/s.
Figure 3.6 Continued.
Figure 3.7 Transformation from pixels to physical coordinates for 30 ft/s. (a) Relative motion (b) histogram.
Figure 3.8 Transformation from pixels to physical coordinates for 45 ft/s. (a) Relative motion (b) histogram.
Figure 3.9 Transformation from pixels to physical coordinates for 60 ft/s. (a) Relative motion (b) histogram
APPENDIX A

MATHCAD SPOT LOCATOR PROGRAM
If the program finds the ending "j-break"

Skip a column if it has a 255 and if the program is searching an ending "j-break"

Begin searching the array by columns

Scale all elements in the array by 255 divided by the maximum element

Open the image file to be analyzed

Define the directory where the files are stored

dist := 
S1 ← "c:\AVI\2_08_2001\aluminum\30fps_2000\"
S3 ← ".bmp"
for  z ∈ 0.. 255
S2 ← num2str (z + 1)
S4 ← concat (S1, S2, S3)
M ← READBMP(S4)
Mmx ← max(M)
numrows ← rows(M)
numcols ← cols(M)
for  i ∈ 0..numcols − 1
for  j ∈ 0..numrows − 1
M_{i,j} ← M_{i,j} \cdot \frac{255}{Mmx}

da ← 0
Jb ← 0
FLAG ← 1
for  j ∈ 0..numcols − 1
for  i ∈ 0..numrows − 1
if  j = numcols − 1 ∧ FLAG = 1 ∧ M_{i,j} = 255
Jb_{a,0} ← j
Jb_{a,1} ← j
FLAG ← 1
if  M_{i,j} = 255 ∧ FLAG = 1
Jb_{a,0} ← j
FLAG ← 0
break
if  FLAG = 0 ∧ j = numcols − 1 ∧ M_{i,j} = 255
Jb_{a,1} ← j
FLAG ← 1
a ← a + 1
break
( break ) if  M_{i,j} = 255 ∧ FLAG = 0
if  FLAG = 0 ∧ i = numrows − 1 ∧ M_{i,j} ≠ 255
Jb_{a,1} ← j − 1
FLAG ← 1
a ← a + 1
break
a ← 0
\begin{verbatim}
Ib ← 0
FLAG ← 1
for i ∈ 0..numrows − 1
  for j ∈ 0..numcols − 1
    if i = numrows − 1 ∧ FLAG = 1 ∧ M_{i,j} = 255
      Ib_{a,0} ← i
      Ib_{a,1} ← i
      FLAG ← 0
    if M_{i,j} = 255 ∧ FLAG = 1
      Ib_{a,0} ← i
      FLAG ← 0
      break
    if FLAG = 0 ∧ i = numrows − 1 ∧ M_{i,j} = 255
      Ib_{a,1} ← i
      FLAG ← 1
      a ← a + 1
      break
    (break) if M_{i,j} = 255 ∧ FLAG = 0
    if FLAG = 0 ∧ j = numcols − 1 ∧ M_{i,j} ≠ 255
      Ib_{a,1} ← i − 1
      FLAG ← 1
      a ← a + 1
      break
   n ← 0
   Ibreak ← Ib
   Jbreak ← Ib
   for i ∈ 0..rows(Ibreak) − 1
     for j ∈ 0..rows(Jbreak) − 1
       subM \_n ← submatrix(M, Ibreak{_{i,0}, Ibreak{_{i,1}, Jbreak{_{j,0}, Jbreak{_{j,1}}
       index{_{n,0} ← Ibreak{_{i,0}
       index{_{n,1} ← Ibreak{_{i,1}
       index{_{n,2} ← Jbreak{_{j,0}
       index{_{n,3} ← Jbreak{_{j,1}
       n ← n + 1
     for i ∈ 0..n − 1
       count ← 0
       sM ← subM_i
                 Repeat re-definition of break variables
                 Begin searching the array by rows
                 If the last row has a 255, and the program is searching for a beginning "i-break".
                 If the program finds a 255 in the array, and the program is searching for an ending "i-break"
                 Skip a row if it has a 255 and if the program is searching an ending "i-break"
                 If the program finds the ending "i-break"
                 Break the array into sub-arrays using the break points
\end{verbatim}
Find the pixel position of the laser.

Find the row and column position of the 255's that make up the laser and store them in an array called pos.

Assign a value of 250 to the noise in the array, so that only the laser is present.

Find the row and column position of the 255's that make up the laser and store them in an array called pos.

Find the pixel position of the laser.

\[
\text{numrows } \leftarrow \text{rows (sM)} \\
\text{numcols } \leftarrow \text{cols (sM)} \\
\text{for } j \in 0..\text{numrows }- 1 \\
\quad \text{for } k \in 0..\text{numcols } - 1 \\
\quad \quad \text{count } \leftarrow \text{count } + 1 \text{ if } sM_{j,k} = 255 \\
\quad \text{cnt}_i \leftarrow \text{count} \\
\quad \text{if } i = 0 \\
\quad \quad \text{indmax } \leftarrow i \\
\quad \quad \text{max } \leftarrow \text{cnt}_i \\
\quad \text{if } i > 0 \land \text{cnt}_i > \text{max} \\
\quad \quad \text{indmax } \leftarrow i \\
\quad \quad \text{max } \leftarrow \text{cnt}_i \\
\text{count } \leftarrow 0 \\
\text{for } i \in 0..n - 1 \\
\quad \text{for } j \in \text{index}_{i,0}..\text{index}_{i,1} \quad \text{if } i \neq \text{indmax} \\
\quad \quad \text{for } k \in \text{index}_{i,2}..\text{index}_{i,3} \\
\quad \quad \quad M_{j,k} \leftarrow 250 \text{ if } M_{j,k} = 255 \\
\quad \quad \text{count } \leftarrow \text{count } + 1 \\
\text{n } \leftarrow 0 \\
\text{pos } \leftarrow 0 \\
\text{for } j \in 0..239 \\
\quad \text{for } k \in 0..319 \\
\quad \quad \text{if } M_{j,k} = 255 \\
\quad \quad \quad x \leftarrow k \\
\quad \quad \quad y \leftarrow j \\
\quad \quad \quad \text{pos}_{n,0} \leftarrow x \\
\quad \quad \quad \text{pos}_{n,1} \leftarrow y \\
\quad \quad \quad n \leftarrow n + 1 \\
\text{ave}_{z,0} \leftarrow \frac{\text{mean (pos}_{\omega})}{\text{ave}_{z,1} \leftarrow \frac{\text{mean (pos}_{\omega})}{\text{refx } \leftarrow \text{ave}_{0,0} \\
\text{refy } \leftarrow \text{ave}_{0,1} \\
\text{dist}_{z,0} \leftarrow \sqrt{(\text{ave}_{z,0} - \text{ave}_{0,0})^2 + (\text{ave}_{z,1} - \text{ave}_{0,1})^2} \quad \text{if } \text{ave}_{z,0} \geq \text{refx} \\
\text{dist}_{z,0} \leftarrow \sqrt{(\text{ave}_{z,0} - \text{ave}_{0,0})^2 + (\text{ave}_{z,1} - \text{ave}_{0,1})^2} \quad \text{otherwise} \\
\text{dist}_{z,1} \leftarrow \text{ave}_{z,0} \\
\text{dist}_{z,2} \leftarrow \text{ave}_{z,1} \\
\text{dist}
Write out the data to a file.

C:\30fps_alum.prn

dist